Financial Regulation in a Quantitative Model of the Modern Banking System*

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Abstract

How does the shadow banking system respond to changes in capital regulation of commercial banks? We propose a quantitative general equilibrium model with regulated and unregulated banks to study the unintended consequences of regulation. Tighter capital requirements for regulated banks cause higher convenience yield on debt of all banks, leading to higher shadow bank leverage and a larger shadow banking sector. At the same time, tighter regulation eliminates the subsidies to commercial banks from deposit insurance, reducing the competitive pressures on shadow banks to take risks. The net effect is a safer financial system with more shadow banking. Calibrating the model to data on financial institutions in the U.S., the optimal capital requirement is around 16%.

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1 Introduction

The goal of the regulatory changes for banks, such as higher capital requirements, after the Great Recession was to reduce the fragility of the financial system. The financial system, however, is complex and includes unregulated financial institutions. So-called shadow banks perform bank-like activities, such as lending and liquidity provision, and compete with traditional banks in these activities. Do tighter regulations on regulated commercial banks cause an expansion of shadow banks? Does a larger shadow banking sector imply an overall more fragile financial system?[1]

In this paper, we build a tractable general equilibrium model to address these questions by quantifying the costs and benefits of tighter bank capital regulation in an economy with regulated commercial banks and unregulated shadow banks. Both bank types provide funding for a bank dependent production sector, financed with equity issued in a competitive market and with deposits issued to households that value their liquidity services. All banks have the option to default and therefore may not repay their depositors. Our model focuses on liability side differences between commercial banks and shadow banks, and assumes that both bank types hold the same assets. Commercial bank deposits are insured and therefore riskfree for depositors. While the government may bail out shadow bank debt, a shadow bank bailout is a random event and not an insurance. That is, shadow bank deposits are in principle uninsured and thus risky for depositors. Because of the lack of full deposit insurance, shadow banks can be subject to bank runs. The deposit insurance for commercial banks gives them a competitive advantage, leading to a larger than socially optimal level of the commercial banking sector and riskier liquidity provision by shadow banks.

Calibrating the model to data from the Flow of Funds, NIPA, Compustat, bank call reports, and to data on interest rates, we show that a higher capital requirement on commercial banks indeed increases the size of the shadow banking sector. The shadow banking sector does indeed become riskier. However, the increase in the risk of the shadow banking sector is economically small and dominated by a large reduction in the riskiness of commercial banks. Thus, an increase in the capital requirement on commercial banks

[1] The example of Regulation Q tells a cautionary tale. Introduced after the Great Depression in the 1930s to curb excessive competition for deposit funds it had little effect on banks as long as interest rates remained low. When interest rates rose in the 1970s, depositors looked for higher yielding alternatives and the competition for their savings generated one: money market mutual funds (Adrian and Ashcraft (2016)). Asset-backed commercial paper conduits are another example for entities that emerged arguably as a response to tighter capital regulation (see Acharya, Schnabl, and Suarez (2013)). These examples highlight the unintended consequences of regulatory policies.

leads to an overall reduction of financial fragility in our model.

The main mechanism for an overall safer financial system despite an expansion of the shadow banking sector is due to a *competition effect*. This effect arises because both types of banks, traditional and shadow, compete for equity capital from household investors, and have to offer the same return on equity in equilibrium. Since liquidity services provided by the debt of shadow and traditional banks are imperfect substitutes, the model predicts a socially optimal share of overall liquidity produced by each type of bank. This is where the competition effect comes in: due to government guarantees, traditional banks have a competitive advantage, attract more equity, and provide a larger than optimal share of liquidity in equilibrium. As a result, shadow liquidity is relatively scarce and earns an inflated convenience yield. Shadow banks react with higher leverage and thus riskiness relative to the constrained efficient allocation. An increase in the capital requirement reduces commercial banks' competitive advantage vis-a-vis shadow banks. In response, shadow banks expand, their debt becomes less scarce, reducing its convenience yield and *increasing* shadow banks' debt financing costs. As a result shadow banks reduce leverage and hence take on less risk.

Our model also captures a countervailing force, which we call the *demand effect*, to the risk dampening competition effect. As long as households demand for aggregate liquidity services is downward sloping, an increase in the capital requirement reduces the supply of commercial bank deposits and raises the convenience yield on both shadow bank and commercial bank debt. This effect *decreases* shadow banks' debt financing costs and provides incentives to increase leverage and take on more risk.

Which effect dominates, depends on the parameters of the model. We find that the demand effect dominates the competition effect and shadow banks increase leverage in response to a higher capital requirement. The risk-dampening force of the competition effect and the large reduction in risk-taking by commercial banks, however, mean that on net the financial system becomes more stable, despite riskier shadow banks. The optimal capital requirement trades-off an increase in consumption driven by a reduction in bankruptcy losses, due to a more stable financial system, against a reduction in liquidity services. Across various parameterizations, the optimal capital requirement is around 16%.

We provide suggestive evidence for our model by comparing the post-crisis period in the data to the model's response to a financial reform that resembles the post-crisis reforms. We model the pre-crisis period as a period with a relatively low capital requirement on commercial banks, a large implicit guarantee for shadow banks, and too optimistic beliefs about a possible run on shadow banks. To capture the financial crisis, we hit the model economy with a large productivity shock and a run on shadow banks. After this shock, the financial system undergoes reforms that result in a small increase in the capital requirement, a reduction in the bailout probability of shadow banks, and correct beliefs about future runs on the shadow banking sector. Our model captures the post-crisis time series pattern of commercial bank leverage and the commercial bank liquidity premium in the data. The reduction in the bailout probability of shadow banks causes a reduction in shadow bank leverage, also in line with the data. Since the reform causes investors to no longer underestimate the risk in the shadow banking system, the shadow banking share in liquidity production shrinks, as in the data. It is important to stress that our definition of the shadow bank share is based on bank liabilities and not based on assets.[2] An important question for future research is to study how our liability-driven mechanism interacts with asset side differences between regulated and unregulated banks.

In sum, our model suggests that a tighter capital requirement for commercial banks will cause a shift towards riskier shadow banks. The increase in risk-taking of shadow banks is coming from the demand effect. Quantitatively, shadow banks' increase in risk-taking is modest due to the competition effect. As a result of having safer commercial banks, the competitive pressure on shadow banks to deliver high equity returns to their investors declines. The net-effect from a reduction in commercial banks' risk-taking, and the slight increase in shadow banks' risk-taking, is a more stable financial system.

Related Literature. Our paper is part of a growing literature at the intersection of macroeconomics and banking that tries to understand optimal regulation of banks in a quantitative general equilibrium framework.[3] Our modeling approach draws on recent work that analyzes the role of financial intermediaries in the macroeconomy and assumes that investors can only access assets through an intermediary.[4] By introducing limited liability and deposit insurance, and by defining the role of banks as liquidity producers, we bridge the gap to a long-standing microeconomic literature on the functions of banks.[5]

[2]Note that other definitions of shadow banks (e.g., Buchak, Matvos, Piskorski, and Seru (2018)) would yield a different post-crisis trend in shadow banking activity.

[3]E.g. Begenau (2020), Christiano and Ikeda (2014), Elenev, Landvoigt, and Van Nieuwerburgh (2018), Gertler, Kiyotaki, and Prestipino (2016), Davydiuk (2017). Nguyen (2014) and Corbae and D'Erasmo (2019) study quantitative models in partial equilibrium.

[4]E.g. Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), Garleanu and Pedersen (2011), Moreira and Savov (2017). In our model, banks raise debt and equity from investors as in Allen et al. (2015).

[5]For an overview of microeconomic models of banking see Freixas and Rochet (1998). Recent theoretical papers focussing on bank capital include Admati et al. (2014), Malherbe (2020) and Harris et al. (2017).

Our goal is to quantify the unintended consequences of regulating commercial banks for financial stability and macroeconomic outcomes. Other papers have addressed closely related questions but not in a quantitative setting.[6] Bengui and Bianchi (2018) provide both a theoretical and quantitative analysis of optimal macroprudential taxation of levered firms when regulators cannot enforce these taxes on a subset of firms. More broadly, the role of shadow banks in the recent financial crisis has motivated a number of papers that propose theories why shadow banks emerged and why they can become unstable (e.g., Gennaioli, Shleifer, and Vishny (2013) and Moreira and Savov (2017)). Shadow banks are often viewed to emerge in response to tighter regulation (e.g., Plantin (2015); Huang (2018); Xiao (2020); Farhi and Tirole (2020)), or because they produce financial services using a different technology compared to traditional banks (e.g., Gertler, Kiyotaki, and Prestipino (2016); Ordoñez (2018); Martinez-Miera and Repullo (2017); Dempsey (2020)). Our paper captures both views as shadow banks can exist independently of how tightly the traditional banking sector is regulated. Yet tighter financial regulation can make the shadow banking sector more attractive and lead to its expansion.[7]

A key difference to other quantitative work is that we explicitly model moral hazard arising from deposit insurance (and more generally government guarantees) akin to Bianchi (2016).[8] In addition, we account for a key institutional feature of financial intermediaries by modeling them with limited liability. Hence our setup allows to study welfare-improving bank regulation in a quantitative framework. Since our focus is on liquidity provision as a fundamental role of banking, we also relate to the literature on the demand for safe and liquid assets,[9] and on the role of financial intermediaries in providing such assets.[10] Pozsar, Adrian, Ashcraft, and Boesky (2012), Chernenko and Sunderam (2014), Sunderam (2015), Adrian and Ashcraft (2016), among others, are recent empirical papers documenting the role of shadow banks for liquidity creation.

After we introduce the quantitative model in Section 2, we discuss its main mechanism using a simplified version in Section 3. In Section 4, we explain how we map the model to the data. Section 5 presents the results and Section 6 concludes.

[6] E.g., Plantin (2015); Huang (2018); Ordoñez (2018); Xiao (2020); Martinez-Miera and Repullo (2017).

[7] The paper by Buchak, Matvos, Piskorski, and Seru (2018) empirically estimates how much of the rise in shadow banking, i.e., Fintech firms, is due to a change in the regulatory system or a change in technology.

[8] In contrast to Bianchi (2016), our focus is not on whether the government guarantee itself is optimal.

[9] E.g. Bernanke (2005), Caballero and Krishnamurthy (2009), Caballero, Farhi, and Gourinchas (2016), Gorton et al. (2012), Krishnamurthy and Vissing-Jorgensen (2012)

[10] There is a large theoretical literature on this subject with seminal papers by Gorton and Pennacchi (1990) and Diamond and Rajan (2001).

2 The Quantitative Model

In this section, we present a tractable general equilibrium framework to study the economic consequences of higher capital requirements. The basic structure includes a discrete time, infinite horizon model with a representative households that owns all financial assets in the economy and a Lucas tree that represents non-bank dependent production. Further, the model features two types of banks, regulated commercial banks (C-banks) and unregulated shadow banks (S-banks) that control the capital stock in the economy and production of the bank dependent production sector. Both C- and S-banks provide liquidity services to households by issuing deposits under limited liability.[11] C-banks benefit from deposit insurance but are also subject to capital requirements. S-banks are fragile and face the risk of large withdrawals (banks runs) due to the lack of deposit insurance. They may also be given a bailout by the government, but this is random.

To write things more compactly, we slightly abuse notation and denote the dependence on the aggregate state vector $\mathcal{Z}_t$ (defined in Section 2.5) with the subscript t. Any additional dependence will be denoted in terms of functions, e.g., $x_t\left(y_t\right)$ means that x depends on the aggregate state vector $\mathcal{Z}_t$ and the additional state variable y_t.

2.1 Preferences

The representative household values consumption and liquidity services:

$$U\left(C_t, H\left(A_t^S, A_t^C\right)\right) = \frac{C_t^{1-\gamma}}{1-\gamma} + \psi \underbrace{\frac{\left[\alpha(A_t^S)^\epsilon + (1-\alpha)(A_t^C)^\epsilon\right]^{\frac{1-\gamma_H}{\epsilon}}}{1-\gamma_H}}_{:\equiv H\left(A_t^S, A_t^C\right)}, \tag{1}$$

where γ is the inverse of the intertemporal elasticity of substitution for consumption. $H(A_S, A_C)$ is the utility from liquidity that is increasing in A_j, $j = S, C$, the quantity of debt of bank type j held by households. The parameter ψ governs the weight of liquidity services relative to the numeraire consumption. The functional form of $H(A_S, A_C)$ implies a constant elasticity of substitution between C-bank and S-bank liquidity, parameterized by $\epsilon \in (-\infty, 1)$, and decreasing returns in overall liquidity, parameterized by $\gamma_H \geq 0$. The parameter α determines the weight each type of liquidity receives in generating aggregate liquidity benefits.

[11] Farhi and Tirole (2020) show how regulated and unregulated banks can both emerge in equilibrium

The liquidity preference specification in (1) implies that (i) households value debt issued by both types of banks beyond its pecuniary payoff, and (ii) S-bank and C-bank debt are imperfect substitutes in liquidity services production, independent of their relative riskiness. Implication (i) follows the literature that models liquidity preferences via a money-in-the-utility specification (see Poterba and Rotemberg (1986)). Feenstra (1986) shows that the reduced-form preference specification is functionally equivalent to microfounding a demand for money with transaction costs. Krishnamurthy and Vissing-Jorgensen (2012) and others argue that this utility specification is consistent with several theories for the valuation of liquidity and safety, for example because assets provide collateral benefits. In our particular case, the liquidity preference function H narrowly reflects the money-like attributes of bank deposits (C-bank debt) or close substitutes such as money market fund shares (S-bank debt). Thus, households value these assets because of their immediate and certain convertibility into a medium of exchange in the sense of e.g. Gorton and Pennacchi (1990).

Both traditional bank deposits and money market accounts are very safe relative to other assets available to households. However, C-bank debt is insured and completely safe, whereas S-bank debt may suffer fractional default. The lower risk of C-bank debt means that households assign a greater liquidity value to it than to S-bank debt, which will be reflected quantitatively through a value of $\alpha < 1/2$ for the relative weight on S-bank liquidity in (1). The specification in (1) assumes that both α and the degree of substitutability ϵ are constant over time and do not depend on fluctuations in riskiness of S-bank debt. In Section 4.4, we will relax this restriction and allow the weight on S-bank liquidity to be a time-varying function of S-bank debt relative to insured deposits at times when S-bank default risk is high or when S-banks are exposed to run risk.

The CES functional form of H can be understood as aggregation of the liquidity demand of households with heterogeneous preferences. This heterogeneity could reflect unmodeled differences in households' financial situation. For instance, the monthly withdrawal limits and lower branch representation associated with many non-commercial bank money market accounts may not be desirable for hand-to-mouth agents facing unexpected expenditure shocks. Such heterogeneous preferences over different types of money-like assets can give rise to the CES form (see e.g., Anderson et al. (1989)).[12]

[12] Alternatively, heterogeneous information about the quality or price of both goods (a commercial bank deposit or a money market account) would allow a microfoundation of the CES liquidity function based on rational inattention with heterogeneous signals such as in Matějka and McKay (2015) and Matveenko (2020).

2.2 Production Technology

There is a continuum of mass one of each type of bank, $j = C, S$. Banks operate a Cobb-Douglas production technology combining capital and labor. That is, similar to Brunnermeier and Sannikov (2014), we assume that banks are directly involved in the production economy. Each bank owns productive capital $\hat{K}_t^j$ at the beginning of the period. We provide details on banks' intertemporal optimization problem and aggregation in Sections 2.3 and 2.4 below. Bank production is exposed to Z_t, an aggregate productivity shock common to all banks. Banks hire labor N_t^j from households at competitive wage w_t and combine it with their capital to produce

$$Y_t^j = Z_t(\hat{K}_t^j)^{1-\eta}(N_t^j)^\eta,$$

where η is the labor share. After production, capital depreciates at rate δ_K. Banks can also invest using a standard convex technology governed by the parameter $\phi_I \geq 0$. Creating I_t^j units of the capital good requires

$$I_t^j + \frac{\phi_I}{2}\left(\frac{I_t^j}{\hat{K}_t^j} - \delta_K\right)^2 \hat{K}_t^j$$

units of consumption. Banks can sell new capital goods and their non-depreciated capital in a competitive market at price p_t. Defining the investment rate $i_t^j = I_t^j/\hat{K}_t^j$ and the labor-capital ratio $n_t^j = N_t^j/\hat{K}_t^j$, we can write the gross payoff per unit of capital as

$$\Pi_t^j = Z_t(n_t^j)^\eta - w_t n_t^j + \left(1 - \delta_K + i_t^j\right) p_t - i_t^j - \frac{\phi_I}{2}(i_t^j - \delta_K)^2,$$

where the first and second term are the revenue from production and the wage bill per unit of capital, the third term denote the proceeds from selling one unit of non-depreciated capital and new capital per unit of capital, while the last two terms denote the expenses for producing new capital per unit of capital.

Households can also hold capital and produce directly. However, as in Brunnermeier and Sannikov (2014) and Gertler et al. (2020), we assume that households operate capital less efficiently, leading to lower productivity $\underline{Z}_t < Z_t$ and a higher depreciation rate $\underline{\delta}_K > \delta_K$. Further, households do not have access to an investment technology. Defining $n_t^H = N_t^H/\hat{K}_t^H$, the gross payoff per unit of capital when operated by households is

$$\Pi_t^H = \underline{Z}_t(n_t^H)^\eta - w_t n_t^H + (1 - \underline{\delta}_K)\, p_t.$$

Labor input and investment decisions Within each period, banks of either type $j = C, S$ choose labor input and investment. The first-order conditions for labor input and investment (see Eq. (33) and (34) in Appendix A.1) allow us to simplify the gross-payoff per unit of capital for banks and households by substituting for the equilibrium wage and investment rate

$$\Pi_t^j = (1-\eta) Z_t (n_t^j)^\eta + p_t - \delta_K + \frac{(p_t - 1)^2}{2\phi_I}, \tag{2}$$

$$\Pi_t^H = (1-\eta) \underline{Z}_t (n_t^H)^\eta + p_t (1 - \underline{\delta}_K). \tag{3}$$

The gross payoffs of capital are a function of the aggregate shocks, the price of capital, and labor-capital ratio.

2.3 S-banks

We now describe the optimization problem of S-banks. Banks make labor input and investment decision and are subject to runs within a period. In addition, S-banks choose the amount of capital to purchase for next period K_{t+1}^S and the amount of deposits to issue to households B_{t+1}^S at price q_t^S. S-bank debt is generally risky for households, but the government decides with probability π_B to bail out defaulting S-bank deposits. We introduce capital adjustment costs on top of investment adjustment costs to capture balance sheet rigidities stemming from illiquid assets.

Bank runs and timing To capture the fragility of S-banks, we introduce bank runs in the S-bank sector similar to Allen and Gale (1994). A fraction of S-bank deposits π_t^R is withdrawn early within a given period (affecting all shadow banks equally), where $\pi_t^R \in \{0, \bar{\pi}^R > 0\}$, following a two-state Markov chain. When deposits are withdrawn, S-banks need to liquidate a fraction of their assets by selling them to households at price Π_t^H defined in Eq. (3). Liquidated assets do not yield any output to the bank. Households sell the assets again in the regular capital market later in the same period.[13] The timing of decisions within each period is as follows:

1. Aggregate shocks Z_t, $\underline{Z}_t$ and π_t^R are realized.
2. If $\pi_t^R = \bar{\pi}^R$, S-banks sell capital worth $\bar{\pi}^R B_t^S$ to households at price Π_t^H.

[13]Since both transactions take place within the same period and households are unconstrained, households' marginal product of capital is Π_t^H that is lower than that of banks, Π_t^j. Hence, households never optimally own any capital at the end of the period.

3. Production of all banks and households and investment decisions of banks ensue.

4. Idiosyncratic payoff shocks of banks are realized. Default decisions.

5. Banks choose their portfolios. Surviving banks pay dividends and new banks are set up to replace liquidated bankrupt banks.

6. Government bails out all defaulting C-bank deposits. S-bank deposits are bailed out with probability π_B.

7. Households consume.

To pay out its depositors in case of a withdrawal shock ($\pi_t^R = \bar{\pi}^R$) at step 2, the fraction of assets that needs to be liquidated is

$$\ell_t^S \equiv \frac{\pi_t^R B_t^S}{K_t^S \Pi_t^H}.$$

Thus, the capital available for production at step 3 is $\hat{K}_t^S \equiv (1 - \ell_t^S) K_t^S$.[14]

Portfolio problem At step 5 of the intraperiod sequence of events, S-banks solve a portfolio choice problem. At this time, S-banks are subject to idiosyncratic payoff shocks $\rho_{t,i}^S \sim F^S$ that are *iid* across banks and over time. We characterize S-banks' portfolio problem recursively. In Appendix A.1, we show that at the time banks choose their new portfolio, all banks have the same value and face the same optimization problem. They choose how much capital to buy for next period, K_{t+1}^S, and how many deposits to issue, B_{t+1}^S to maximize current period dividend payout to shareholders and the continuation value. To save notation, we make use of the fact that all S-banks face the same optimization problem and omit individual subscript i from the presentation of the bank problem. Note that we introduce intertemporal balance sheet rigidities by subjecting banks' choice of K_{t+1}^S to a quadratic adjustment cost. The total dividend the representative S-bank pays to its shareholders at step 5 is given by

$$D_t^S = \rho_t^S \Pi_t^S \hat{K}_t^S - (1 - \pi_t^R) B_t^S + q_t^S \left(B_{t+1}^S, K_{t+1}^S \right) B_{t+1}^S - p_t K_{t+1}^S - \frac{\phi_K}{2} \left(\frac{K_{t+1}^S}{\hat{K}_t^S} - 1 \right)^2 \hat{K}_t^S. \quad (4)$$

The first term denotes the payoff of capital after the realization of the S-bank specific *iid* capital payoff shock ρ_t^S. The second term denotes deposit repayment obligations that

[14] In our baseline calibration as well as all other numerical experiments we consider. $\bar{\pi}^R$ is always below $K_t^S \Pi_t^H / B_t^S$ so that banks can always redeem early withdrawals.

remained after the realization of the run shock. The third term denotes new funds from deposits issuance at price $q_t^S\left(B_{t+1}^S, K_{t+1}^S\right)$, and the fourth term is new capital purchased at price p_t. The last term denotes the balance sheet adjustment costs.

We characterize the S-bank's portfolio problem recursively using the value function $\hat{V}_t^S(\hat{K}_t^S, \rho_t^S)$. Recall that in addition to the two individual state variables, the post-run capital stock $\hat{K}_t^S$ and the payoff shock ρ_t^S, the bank's value is indexed by t and thus depends on the aggregate state vector $\mathcal{Z}_t$. The value of surviving S-bank is

$$\hat{V}_t^S(\hat{K}_t^S, \rho_t^S) \quad = \quad \max_{K_{t+1}^S, B_{t+1}^S} D_t^S \;+\; \mathrm{E}_t\left[M_{t,t+1} \max\left\{\hat{V}_{t+1}^S(\hat{K}_{t+1}^S, \rho_{t+1}^S), V_{t+1}^{S,Def}\right\}\right], \quad (5)$$

where $M_{t,t+1}$ is the stochastic discount factor of households and $V_{t+1}^{S,Def} = -\delta_S \Pi_{t+1}^S \hat{K}_{t+1}^S$ is the value of default with default utility-penalty parameter $\delta_S \geq 0$. The default penalty is proportional to the asset value, which retains the problem's homogeneity in capital $\hat{K}_t^S$.

To simplify the optimization problem further, we recognize that profits from real production activities $\rho_t^S \Pi_t^S \hat{K}_t^S$ and deposit obligations $(1-\pi_t^R)B_t^S$ are irrelevant for banks' portfolio choice after they have decided not to default, i.e., after step 4 of the time line above. Hence, all banks face the same portfolio choice problem for period $t+1$, conditional on having the same capital $\hat{K}_t^j$. This allows us to define a new value function $V_t^S(\hat{K}_t^S) = \hat{V}_t^S(\hat{K}_t^S, \rho_t^S) - \rho_t^S \Pi_t^S \hat{K}_t^S + (1-\pi_t^R)B_t^S$ such that we can rewrite (5) as

$$\begin{aligned} V_t^S(\hat{K}_t^S) = \max_{K_{t+1}^S, B_{t+1}^S} q_t^S\left(B_{t+1}^S, K_{t+1}^S\right) B_{t+1}^S - p_t K_{t+1}^S - \frac{\phi_K}{2}\left(\frac{K_{t+1}^S}{\hat{K}_t^S} - 1\right)^2 \hat{K}_t^S + \\ \mathrm{E}_t\left[M_{t,t+1} \max\left\{\rho_{t+1}^S \Pi_{t+1}^S \hat{K}_{t+1}^S - B_{t+1}^S(1-\pi_{t+1}^R) + V_{t+1}^S(\hat{K}_{t+1}^S), V_{t+1}^{S,Def}\right\}\right]. \end{aligned} \quad (6)$$

Two properties of the S-bank problem allow us to obtain aggregation. First, idiosyncratic profit shocks ρ_t^S are uncorrelated over time. Second, the value function is homogeneous in capital. We use these properties to write the bank value function in terms of the value per unit of capital $v_t^S = V_t^S(\hat{K}_t^S)/\hat{K}_t^S$, which only depends on the aggregate state vector $\mathcal{Z}_t$. The two intertemporal choices are the deposit-capital ratio $b_{t+1}^S = \frac{B_{t+1}^S}{K_{t+1}^S}$ and capital growth $k_{t+1}^S = \frac{K_{t+1}^S}{\hat{K}_t^S}$. We further define bank leverage as

$$L_{t+1}^S \equiv \frac{B_{t+1}^S}{\Pi_{t+1}^S K_{t+1}^S} = \frac{b_{t+1}^S}{\Pi_{t+1}^S},$$

with Π_t^S being the effective payoff per unit of capital as defined in (2). Using this definition, we write the S-bank's problem as[15]

$$v_t^S = \max_{b_{t+1}^S \geq 0, k_{t+1}^S \geq 0} \underbrace{-\left(k_{t+1}^S\left(p_t - q_t^S\left(b_{t+1}^S\right)b_{t+1}^S\right) + \frac{\phi_K}{2}\left(k_{t+1}^S - 1\right)^2\right)}_{\text{cost of portfolio for } t+1} \tag{7}$$

$$+ k_{t+1}^S \underbrace{\mathrm{E}_t\left[M_{t,t+1}\,\Pi_{t+1}^S \max\left\{\left(1-\ell_{t+1}^S\right)\left(\rho_{t+1}^S + \frac{v_{t+1}^S}{\Pi_{t+1}^S}\right) - L_{t+1}^S(1-\pi_{t+1}^R), -\delta_S\left(1-\ell_{t+1}^S\right)\right\}\right]}_{\text{expected payoff per levered unit of capital in } t+1}.$$

S-bank equity owners optimally trade off the cost of investing in the bank's portfolio today against the expected payoff next period. They internalize that the price of their debt, q_t^S, is a function of their default risk and thus their capital structure. The max-operator in the expectation on the RHS reflects the continuation value per unit of levered capital, taking into account the optimal default decision next period and the possibility of an early withdrawal shock that forces the bank to sell fraction ℓ_{t+1}^S of its capital to households.[16] Eq. (7) clarifies that S-banks optimally default at step 4 in the intraperiod time line when $\rho_t^S < \hat{\rho}_t^S$, with

$$\hat{\rho}_t^S = \frac{(1-\pi_t^R)L_t^S - (1-\ell_t^S)\left(\frac{v_t^S}{\Pi_t^S} + \delta_S\right)}{1-\ell_t^S}. \tag{8}$$

The probability of default is thus $F_{\rho,t}^S \equiv F^S\left(\hat{\rho}_t^S\right)$. We provide more details on how to aggregate the problem of banks in general in Appendix A.1 and derive Euler equations for S-banks in Appendix A.4.

2.4 C-banks and Government

C-banks. C-banks differ from S-banks in four ways: (i) they issue short-term debt that is insured and hence risk free for creditors, (ii) they do not experience runs (as result of (i)), (iii) they are subject to a capital requirement, and (iv) they pay an insurance fee of κ

[15] Homogeneity of the value function $V_t^S(\hat{K}_t^S)$ of degree one in capital requires that the debt price function $q_t^S\left(B_{t+1}^S, K_{t+1}^S\right)$ is jointly homogeneous of degree zero in B_{t+1}^S and K_{t+1}^S. This property is satisfied, as the price function only depends on the ratio $b_{t+1}^S = \frac{B_{t+1}^S}{K_{t+1}^S}$, which can be verified from the household first-order condition for S-bank debt in equation (23), Appendix A.3.

[16] For a hypothetical bank without default risk, early withdrawal shocks and leverage, this continuation value would simply be $\rho_{t+1}^S \Pi_{t+1}^S + v_{t+1}^S$.

for each unit of debt they issue. Using the same notation as for S-banks, C-banks solve

$$v_t^C = \max_{b_{t+1}^C \geq 0, k_{t+1}^C \geq 0} - \left(k_{t+1}^C \left(p_t - \left(q_t^C - \kappa \right) b_{t+1}^C \right) + \frac{\phi_K}{2} \left(k_{t+1}^C - 1 \right)^2 \right)$$
$$\mathrm{E}_t \left[M_{t,t+1} \, k_{t+1}^C \, \Pi_{t+1}^C \max \left\{ \rho_{t+1}^C + \frac{v_{t+1}^C}{\Pi_{t+1}^C} - L_{t+1}^C \, , -\delta_C \right\} \right], \tag{9}$$

subject to the capital requirement

$$(1-\theta) p_t \geq b_{t+1}^C. \tag{10}$$

C-banks optimally default at step 4 in the intraperiod time line when $\rho_t^C < \hat{\rho}_t^C$, with

$$\hat{\rho}_t^C = L_t^C - \frac{v_t^C}{\Pi_t^C} - \delta_C, \tag{11}$$

where $\delta_C \geq 0$ is a default penalty parameter. Given $\rho_t^C \sim F^C$, the probability of default is $F_{\rho,t}^C \equiv F^C \left(\hat{\rho}_t^C \right)$. The full optimization problem of C-banks including Euler equations is in Appendix A.5.

Bankruptcy, Bailout and Government Budget Constraint. If a bank declares bankruptcy, its equity (and continuation value) becomes worthless, and creditors seize all of the banks assets, which are liquidated. The recovery amount per unit of debt issued is

$$r_t^j = (1-\xi^j) \frac{\rho_t^{j,-} \left(1 - \ell_t^j \right)}{L_t^j \left(1 - \pi_t^R I_{j=S} \right)},$$

for $j = S, C$ and $I_{j=S}$ is an indicator function that takes the value of 1 if $j = S$ else it is 0. A fraction ξ^j of assets is lost in the bankruptcy proceedings, with $\rho_t^{j,-} \equiv \mathrm{E} \left(\rho_t^j \,|\, \rho_t^j < \hat{\rho}_t^j \right)$ being the average idiosyncratic shock of defaulting banks. Since C-banks do not experience runs, $\ell_t^C = 0 \; \forall t$. Bankruptcy losses are real losses to the economy. They reflect both greater capital depreciation of foreclosed banks, and real resources destroyed in the bankruptcy process that reduce bank profits.

After the bankruptcy proceedings are completed, a new bank is set up to replace the failed one. This bank sells its equity to new owners, and is otherwise identical to a surviving bank after asset payoffs.

If a S-bank defaults, the recovery value per unit of debt is used to pay the claims of creditors to the extent possible. We further consider the possibility that the government bails out the creditors of the defaulting S-bank with probability π_B, known to all agents ex-ante. If a C-bank declares bankruptcy, it is taken over by the government that uses lump-sum taxes and revenues from deposit insurance, κB^C_{t+1}, to pay out the bank's creditors in full. Summing over defaulting C-banks and S-banks that are bailed out, we define lump sum taxes as

$$T_t = F^C_{\rho,t}\left(1 - r^C_t\right) B^C_t - \kappa B^C_{t+1} + \pi_B F^S_{\rho,t}\left(1 - r^S_t\right)\left(1 - \pi^R_t\right) B^S_t.$$

2.5 Households and Equilibrium

Households. Each period, households receive an endowment from a Lucas tree Y_t and the payoffs from owning all equity and debt claims on intermediaries, yielding financial wealth W_t. They further inelastically supply their unit labor endowment at wage w_t and pay lump-sum taxes T_t.[17] Households choose consumption C_t, deposits of both banks for redemption next period, A^S_{t+1} and A^C_{t+1}, and bank equity purchases S^S_t and S^C_t, to maximize utility (1) subject to their intertemporal budget constraint

$$W_t + Y_t + w_t - T_t \geq C_t + \sum_{j=S,C} p^j_t S^j_t + \sum_{j=S,C} q^j_t A^j_{t+1}, \tag{12}$$

where p^j_t, $j = S, C$, denotes the market price of bank equity of type j. The transition law for household financial wealth W_t is

$$\begin{aligned} W_{t+1} = &\sum_{j=S,C} (1 - F^j_{\rho,t+1})\left(D^{j,+}_{t+1} + p^j_{t+1}\right) S^j_t \\ &+ \left(1 - \pi^R_{t+1}\right) A^S_{t+1}\left[1 - F^S_{\rho,t+1} + F^S_{\rho,t+1}\left(\pi_B + (1 - \pi_B)\, r^S_{t+1}\right)\right] + \pi^R_{t+1} A^S_{t+1} \\ &+ A^C_{t+1}, \end{aligned}$$

where $D^{j,+}_{t+1}$ is the dividend of banks of type $j = S, C$ conditional on survival as defined in the Appendix A.3. This appendix section states the full optimization problem of households including their optimality conditions.

Equilibrium. The aggregate state vector $\mathcal{Z}_t$ consists of the exogenous productivity shocks driving Y_t and Z_t, the aggregate capital holdings of each type of bank K^j_t, for $j = S, C$, and

[17] Households also receive production income $\Pi^H_t \hat{K}^H_t$, which is equal to capital purchases from banks. Thus, these two terms net to zero.

deposit holdings at the beginning of period, A_t^j, for $j = S, C$. Market clearing requires that within period capital holdings by households are $\hat{K}_t^H = \ell_t K_t^S$, and that households purchase all securities issued by banks, which implies $B_{t+1}^j = A_{t+1}^j$, for $j = S, C$, in deposit markets, and $S_t^j = 1$ in equity markets. Labor supply by households has to equal labor demand by banks, and by producing households in case of fire sales, implying $N_t^S + N_t^C + N_t^H = 1$. We provide a formal equilibrium definition as well as market clearing conditions for capital and consumption in Appendix A.2. In the capital market, bank failures lead to endogenous depreciation in addition to production-induced depreciation δ_K. Similarly, bank failures also cause a loss of resources in the goods market. Appendix I.a lists the full set of equations characterizing the equilibrium.

3 Main Mechanism In A Simplified Two-period Model

Before describing the calibration strategy (see Section 4), we discuss the main intuition of the model. To this end, we strip down the quantitative model to its core and focus on a two-period version (times 0 and 1) that we can solve by hand.

3.1 Simple Model Set-up

As before, there is a representative household and two types of banks, C-banks and S-banks. We simplify the production process as follows. The total supply of capital is fixed at unity. Banks buy capital at time 0 at price p in a competitive market. Each unit of capital produces one unit of the consumption good at time 1. As in the quantitative model, banks are financed with equity and uncontingent debt issued to households. Deposit insurance gives C-banks a competitive advantage.

Households are endowed with 1 unit of the capital good at time 0. Their simplified preferences are

$$U = C_0 + \beta \left(C_1 + \psi H(A_S, A_C) \right), \tag{13}$$

where $H(A_S, A_C)$ is the utility from liquidity, and A_j, $j = S, C$ is the quantity of debt of bank type j held by households.

S-banks and C-banks issue debt B_j at prices q_j and equity shares S_j at prices p_j in competitive markets to households. For both type of banks, we will call debt deposits. Both types of banks have limited liability and make optimal default decisions at time 1. In case of default, bank equity becomes worthless. We also assume that all S-bank or

C-bank assets are lost in default and cannot be used to repay depositors. Yet, C-bank deposit insurance makes C-bank deposits perfectly safe for depositors. When C-bank equity is insufficient to fully repay depositors, the government makes up the shortfall by raising lump-sum taxes on households. In contrast, deposits issued by S-banks are risky. Since we assumed no recovery in case of default, any return on S-bank capital is lost and depositors lose all their deposits when the S-bank defaults.

S-banks Problem. An S-bank chooses how much capital, K_S, at price p to buy and how many deposits, B_S, to issue to raise $q_S B_S$ at time 0. It needs to raise the difference in initial equity from households. As in the quantitative model, individual banks receive idiosyncratic production shocks ρ_S at time 1 that are distributed *iid*, such that the total payoff to capital at time 1 is $\rho_S K_S$. S-banks' maximization problem is a simplified version of S-banks' problem in the full model (see Eq. 7) without adjustment costs, runs, and a default penalty. Thus, each S-bank maximizes its expected net present value

$$\max_{K_S \geq 0, B_S \geq 0} \quad \underbrace{q_S(B_S, K_S)B_S - pK_S}_{\text{equity raised at } t=0} + \beta \mathrm{E}\left[\underbrace{\max\left\{\rho_S K_S - B_S, 0\right\}}_{\text{dividend paid at } t=1}\right]. \tag{14}$$

The price for S-banks' deposits, $q_S(B_S, K_S)$, depends on the leverage choice of each bank. Households take into account that higher bank leverage increases the probability of a default. The bank internalizes this effect when making its leverage decision.

C-bank Problem. C-banks differ from S-banks in two ways. First, C-banks issue safe deposits due to the government guarantee (deposit insurance). Hence the price at which they raise deposits, q_C, is not sensitive to their leverage choice. Secondly, C-banks are subject to a regulatory capital constraint that limits the amount of deposits they can issue to a fraction $1-\theta$ of the expected payoff of capital at time 1, $\mathrm{E}(\rho_C K_C)$. C-banks solve

$$\max_{K_C \geq 0, B_C \geq 0} \quad \underbrace{q_C B_C - pK_C}_{\text{equity raised at } t=0} + \beta \mathrm{E}\left[\underbrace{\max\left\{\rho_C K_C - B_C, 0\right\}}_{\text{dividend paid at } t=1}\right], \tag{15}$$

subject to the equity capital requirement

$$B_C \leq (1-\theta)\mathrm{E}(\rho_C K_C). \tag{16}$$

Bank Size and Leverage Choices. To solve the bank problem, we use the fact that it is homogeneous in capital and that the ρ shocks are *iid* across banks. We can then divide the bank objective by K_j and separate each bank's problem into a size (K_j) decision and a leverage ($L_j = B_j/K_j$) decision. The expected dividend at $t = 1$ becomes

$$\mathrm{E}\left[\max\left\{\rho_j - L_j, 0\right\}\right] = \left(1 - F_j(L_j)\right)\left(\rho_j^+ - L_j\right),$$

where we have defined the conditional expectation $\rho_j^+ = \mathrm{E}(\rho_j\,|\rho_j > L_j)$.[18]

Households. Households are endowed with one unit of capital. They optimally sell this capital to banks at price p. They buy deposits A_j and equity shares S_j of bank type j at time 0, such that their time 0 budget constraint is

$$C_0 = p - q_S A_S - q_C A_C - p_S S_S - p_C S_C. \tag{17}$$

Time-1 consumption is therefore

$$C_1 = (1 - L_S)A_S + A_C + S_S K_S\left(1 - F_S(L_S)\right)\left(\rho_S^+ - L_S\right) + S_C K_C\left(1 - F_C(L_C)\right)\left(\rho_C^+ - L_C\right) - T, \tag{18}$$

where T denotes government lump-sum taxation to bail out deposits, i.e., $T = L_C B_C$, $(1 - L_S)A_S$ and A_C are deposit redemptions for S- and C-banks, respectively. The terms $\left(1 - F_j(L_j)\right)\left(\rho_j^+ - L_j\right)$ denote the expected cash-flow from owning bank type j equity. Households choose C_0, C_1, S_j, and A_j, $j = C, S$ to maximize utility (13) subject to constraints (17) and (18).

Equilibrium definition. The equilibrium is a set of prices $\{p, q_S, q_C, p_S, p_C\}$ and quantities $\{C_0, C_1, K_S, K_C, L_S, L_C, S_S, S_C, A_C, A_S\}$, such that households maximize (13) subject to constraints (17) and (18), S-banks maximize (14), C-banks maximize (15) subject to (16), and the markets for capital $1 = K_S + K_C$, equity shares (sum to 1) and deposits of both bank types, $A_j = B_j$, clear. More details are in Appendix II.a.

3.2 Efficient Allocation versus Competitive Equilibrium

Efficient allocation. To understand under which conditions higher capital requirements improve welfare, we first solve for the optimal allocation of capital and leverage of each

[18] Appendix II.c separates out the size and leverage optimization problem for both banks.

type of bank from the perspective of a social planner that maximizes household welfare. The planner is restricted to the same resource constraint as the decentralized economy, and has to use the same risky intermediation technology to produce liquidity services. Therefore, numeraire consumption in periods 0 and 1 is restricted by the resource constraints of the decentralized economy in equations (1) and (2) in Appendix II.a. Recall from Eq. (13) that households value liquidity services from holding deposits. We assume that $H(A_S, A_C)$ has the same functional form as in Eq. (1).

Assumption 1.

$$H(A_S, A_C) = \frac{\left(\alpha A_S^{\epsilon} + (1-\alpha) A_C^{\epsilon}\right)^{\frac{1-\gamma_H}{\epsilon}}}{1-\gamma_H}, \tag{19}$$

where the parameters are defined below Eq. 1.

Writing deposits as $A_S = L_S K_S$ and $A_C = L_C K_C = L_C(1-K_S)$, the planner's optimization problem is

$$\max_{K_S, L_S, L_C} \; K_S(1-F_S(L_S))\rho_S^{+} + (1-K_S)(1-F_C(L_C))\rho_C^{+} + \psi H\left(L_S K_S, L_C(1-K_S)\right). \tag{20}$$

The first two terms are time-1 consumption (see Eq. (2) in the Appendix). Proposition 1 characterizes the solution to this problem.

Proposition 1. *If the bank-idiosyncratic shocks ρ_j, for $j = S, C$, are drawn from the same distribution, the optimal ratio of S-bank and C-bank capital is given by*

$$A^* \equiv \frac{A_S}{A_C} = \frac{K_S}{K_C} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\epsilon}}. \tag{21}$$

Optimal leverage is equalized across bank types and given by $L_S = L_C = L^*$, where L^* is a function of parameters and given in the appendix.

Proof. See appendix II.d. □

The allocation of capital reflects the weight each type of liquidity receives in the utility function, parameterized by α. A higher elasticity of substitution $1/(1-\epsilon)$ "tilts" the optimal allocation towards the bank type that receives a greater weight. Leverage is chosen such that the marginal losses from bank defaults equal the marginal utility of liquidity for each type. Since both banks have equally good technologies for producing liquidity of their own type for a given unit of capital, the planner chooses equal leverage for both.

Competitive Equilibrium Characterization. This section introduces one of the two key equilibrium forces: the *competition effect*. This effect arises simply because markets for equity and debt of both banks, as well as for physical capital, are perfectly competitive. Thus, resources must be allocated to both types of banks in equilibrium such that households are indifferent on the margin between investing in either bank type's equity.[19] Since prices of banks' debt directly affect their equity values (see Eqs. (14) and (15)), the requirement of equal equity valuation feeds back to bank leverage and capital purchase choices. To see how the competition effect works, we first need to understand how households price deposits of C- and S-banks. We denote the partial derivatives of the liquidity utility function with respect to the two types of liquidity as $\mathcal{H}_j(A_S, A_C) = \partial H(A_S, A_C)/\partial A_j$, for j=$S, C$, respectively. The household's first-order conditions for S-bank and C-bank are

$$q_C = \beta(1 + \psi\mathcal{H}_C(A_S, A_C)), \tag{22}$$
$$q_S = \beta(1 - F_S(L_S) + \psi\mathcal{H}_S(A_S, A_C)). \tag{23}$$

Eq. (22) and (23) state that the prices of C-bank and S-bank debt, q_C and q_S, must equal their respective expected discounted payoffs plus the discounted marginal liquidity benefit $\beta\psi\mathcal{H}_j(A_S, A_C)$, for j=$S, C$. Since C-bank debt is insured, the expected payoff is unaffected by C-bank default and hence risk free.[20] In contrast, the expected payoff to uninsured S-bank debt is $1 - F_S(L_S)$, reflecting that in expectation a fraction $F_S(L_S)$ of S-banks defaults with a recovery value of zero.

This differential debt pricing affects the optimal leverage and capital choices of both types of banks, with details given in Appendix II.c. In particular, since C-banks can issue insured debt that also generates utility for households, there is no interior optimum to their capital structure choice, and the constraint (16) is always binding. S-banks' leverage choice, on the other hand, trades off the marginal benefit of S-bank liquidity to households against their expected cost of default, which is increasing in leverage.

Furthermore, because of constant returns to scale and competitive markets, both types of banks must have zero expected value in equilibrium.[21] This leads to the following

[19] Allen, Carletti, and Marquez (2015) analyze a similar mechanism in a model in which banks compete with public firms (that also borrow from banks) for equity funds from the same investors. More generally, our framework differs from the canonical macro-finance model with levered intermediaries of e.g. Gertler and Kiyotaki (2010) in that our banks raise deposits *and* equity from outside investors.

[20] We take limited liability and deposit insurance for C-banks, i.e. basic institutional features of the banking system, as given, with the reasons for their existence outside the model. In the quantitative version of the model, we take into account that S-banks face the risk of large withdrawals (banks runs) due to the lack of deposit insurance and may also be given a bailout by the government.

[21] The market value of equity at time 0 must equal to expected payoff of the bank's portfolio at time 1.

capital demand conditions for C-banks and S-banks, respectively:

$$p = \beta\left((1 - F_C(L_C))\rho_C^+ + \psi L_C \mathcal{H}_C(A_S, A_C) + F_C(L_C)L_C\right), \tag{24}$$
$$p = \beta\left((1 - F_S(L_S))\rho_S^+ + \psi L_S \mathcal{H}_S(A_S, A_C)\right). \tag{25}$$

The different debt financing costs for C-banks and S-banks translate into different demands for intermediated capital. Both type of banks value capital for its expected payoff in case of no default, $(1 - F_j(L_j))\rho_j^+$, and its collateral value for the production of liquidity services that households value, $\psi L_j \mathcal{H}_j(A_S, A_C), \forall j \in \{C, S\}$. C-banks assign *additional* value to debt financing, $F_C(L_C)L_C$, since their debt is insured by the government and thus its price is insensitive to C-bank default risk. This additional debt advantage increases C-bank demand for capital that serves as collateral for debt. By equating and simplifying the capital demand conditions (24) – (25), we get the capital market condition

$$\underbrace{(1 - F_S(L_S))\rho_S^+ - (1 - F_C(L_C))\rho_C^+}_{\text{payoff difference}} + \underbrace{\psi\left(L_S \mathcal{H}_S(A_S, A_C) - L_C \mathcal{H}_C(A_S, A_C)\right)}_{\text{liquidity premium difference}} = F_C(L_C)L_C. \tag{26}$$

Since C-banks enjoy the implicit subsidy of government-insured debt (RHS of (26)), S-banks have to compensate in order to be competitive. They can do this either through higher payoffs, or a higher equilibrium liquidity benefit. This equation is at the heart of the competition effect. To see how this relates to capital requirements, notice that C-bank leverage is directly determined by the capital constraint, $L_C = \mathrm{E}(\rho_C)(1 - \theta)$. Thus, the benefit C-banks derive from insured deposits, $F_C(L_C)L_C$, decreases in the capital requirement θ. To understand the competition effect, consider an increase in θ that lowers C-bank leverage. For a given allocation of capital and S-bank leverage, C-banks become less profitable as a result of fewer insured deposits. As a result, investors (i.e., households) shift towards S-bank equity. The S-bank sector expands by purchasing more capital, which raises S-bank liquidity A_S and reduces C-bank liquidity A_C. Since preferences have decreasing returns in each type, S-banks' convenience yields decline and C-banks' rise. This process continues until condition (26) holds again.

Efficiency Properties of Equilibrium. To understand the difference between the planner allocation and the competitive equilibrium, we index equilibria by the liquidity wedge

$m > -1$, where m is implicitly defined by

$$L_C f_C(L_C) = (1+m)\psi\mathcal{H}_C(A_S, A_C), \tag{27}$$

with f_C being the density function of distribution F_C.[22] The factor m represents the wedge between the social marginal benefit of C-bank liquidity $\psi\mathcal{H}_C(A_S, A_C)$, and the marginal cost to society of producing this liquidity $L_C f_C(L_C)$. The social planner solution requires that the costs and benefits of liquidity provision are equal, in which case $m = 0$. In the competitive equilibrium with limited liability and deposit insurance for C-bank deposits, the default risk of C-banks is unpriced. A high value of m implies that C-banks overproduce liquidity in the sense that $L_C f_C(L_C) > \psi\mathcal{H}_C$.

We make the following assumption for analytical tractability.

Assumption 2. *The bank-idiosyncratic shocks ρ_j, for $j = S, C$ are distributed i.i.d Uniform$[0,1]$.*

Given this assumption, we can solve the decentralized equilibrium as a function of m.

Proposition 2. *For any competitive equilibrium,*

(i) S-bank leverage is greater than its social planner solution,

(ii) the S-bank market shares in the debt and capital markets are given by

$$\frac{A_S}{A_C} = \left(\frac{1}{\mathcal{M}}\right)^{\frac{1}{1-\epsilon}} A^* \text{ and } \frac{K_S}{K_C} = (1+m)\left(\frac{1}{\mathcal{M}}\right)^{\frac{2-\epsilon}{1-\epsilon}} A^*, \tag{28}$$

where $\mathcal{M} = \sqrt{(1+m)(3+m)}$ and A^* is defined in Prop. 1.

(iii) there is no $\theta \in [0,1]$ that implements the planner allocation from Proposition 1.

Proof. See appendix II.d. □

Proposition 2 shows that the competitive equilibrium deviates from the planner solution both in terms of leverage choices and capital allocation. In particular, the relative size of the S-bank sector is distorted when compared to the planner solution where $K_S/K_C = A_S/A_C = A^* = (\alpha/(1-\alpha))^{1/(1-\epsilon)}$.

[22] If an equilibrium exists, it is unique.

Since the capital constraint of C-banks is always binding with $L_C = \mathrm{E}(\rho_C)(1-\theta)$, and the C-bank default probability increases in leverage, the regulator can choose θ such that $m = 0$ and C-banks produce liquidity services efficiently given their scale. Part (iii) of Proposition 2 states that even in such a case, the competitive equilibrium does not achieve overall efficiency, because S-banks' share in liquidity provision is too low relative to the social planner solution. The reason is competition between S- and C-banks, as formally expressed by condition (26). Bank equity investors must be indifferent between investing in C-banks or S-banks. The fact that C-banks can issue insured deposits while S-banks cannot, provides C-banks with a competitive advantage. As a result, the C-bank sector is too large (by factor $3^{\frac{1}{2(1-\epsilon)}}$ at $m = 0$). Since the S-bank sector is too small, S-bank liquidity A_S is relatively scarce and S-bank debt enjoys a large liquidity premium. This large premium boosts the profitability of S-banks to the same level as that of C-banks, and also causes S-banks to raise leverage above the value in the planner solution.[23] Thus, absent additional policy tools to "regulate" S-banks, the capital requirement θ is not sufficient to achieve overall efficiency.

3.3 The Effect Of A Higher C-bank Capital Requirement

To see how a higher capital requirement affects the economy, we study the comparative statics of the competitive equilibrium with respect to θ.

Proposition 3. *1. Holding constant all other parameters, an increase in the requirement θ*

(i) reduces C-bank leverage,

(ii) causes an expansion in the S-bank share: $\frac{d(A_S/A_C)}{d\theta} > 0$ and $\frac{d(K_S/K_C)}{d\theta} > 0$,

(iii) can either raise or lower S-bank leverage, depending on model parameters.

2. For $m \geq 0$, a marginal increase in the capital requirement improves aggregate welfare.

Proof. See appendix II.d. □

Part 1(i) follows from the fact that a higher θ tightens the C-bank leverage constraint. Part 1(ii) builds on the results in Proposition 2 to show that a higher capital requirement always leads to a relative increase in the size of the S-bank sector. A tighter capital requirement means that C-banks benefit less from the implicit subsidy of deposit insurance.

[23] S-banks choose leverage efficiently given the level of their liquidity premium $\psi\mathcal{H}_S(A_S, A_C)$. However, since the S-bank sector is too small, the premium is too large.

Thus, they become relatively less profitable and investor equity flows into the shadow banking sector, causing S-banks to expand their market share.

While unambiguously increasing the size of S-banks, Part 1(iii) states that a higher capital requirement has an ambigious effect on S-bank leverage. When the regulator raises θ, there are two effects, the *competition* effect explained above and the *demand* effect. The competition effect underlies the results stated in Part 1(ii) of Proposition 3. Tighter regulation reduces C-banks' competitive advantage stemming from deposit insurance, and thus makes S-banks relatively more competitive. Everything else equal, this effect *lowers* optimal S-bank leverage. However, since the economy features decreasing returns in overall liquidity provision, tightening C-banks' constraint will generally cause an increase in the liquidity premium of both types of banks. To see why, note that the economy has a downward-sloping demand curve for liquidity. An increase in θ shifts the liquidity supply curve to the left and thus leads to higher prices, i.e. liquidity premia. Everything else equal, this *demand* effect causes S-banks to increase leverage. Which effect dominates depends on the parameters of the model. But the following corollary states a special case.

Corollary 1. *If aggregate liquidity production has constant returns to scale ($\gamma_H = 0$), an increase in the capital requirement θ causes lower S-bank leverage.*

If $\gamma_H = 0$, households are perfectly elastic with respect to total liquidity $H(A_S, A_C)$, holding fixed its composition. In that case, the demand effect is zero (the demand curve is flat) and we get $dL_S/d\theta < 0$ due to the competition effect alone.

Implication for the optimal level of capital requirements. Part (2) of Proposition 3 provides a sufficient condition under which an increase in θ improves welfare. In any equilibrium with $m \geq 0$, C-banks (weakly) overproduce liquidity, and raising θ will shrink the wedge m towards zero. Even at $m = 0$, a marginal increase in θ is still unambiguously welfare-improving. The reason is once more competition between both types of banks. In the decentralized equilibrium, the derivative of households' utility with respect to θ

$$\frac{dU(\theta)}{d\theta} = m\mathrm{E}(\rho_C)\psi\mathcal{H}_C(A_S, A_C)K_C + \frac{dK_S}{d\theta}F_C(L_C)L_C, \tag{29}$$

illustrates the trade-off regulators face when setting the optimal capital requirement (see Appendix II.d for the derivation). The first term reflects the standard trade-off that would arise in a model with only C-banks. If $m \geq 0$, C-banks are overproducing liquidity and the term is positive. In a world without S-banks, the optimal level of the capital requirement trades off the increase in consumption due to fewer defaults against the reduction in liquidity provision for C-banks only. In such a model, the second term of (29) would not exist and the optimal θ would simply set $m = 0$. However, the second term reflects the benefit of an expansion in the size of the shadow banking sector, which is too small because of C-banks' competitive advantage. From part 1(ii) of the proposition, we know that raising θ will cause an expansion in the S-bank share, $\frac{dK_S}{d\theta} > 0$, moving the allocation of capital closer to the planner solution. The trade-off of a higher capital requirement in this model is as follows. To allow an expansion of the S-bank share, the regulator raises the capital requirement to reduce the competitiveness of C-banks. At the optimal θ that sets the derivative in (29) to zero, the planner trades off *underproduction* of liquidity from C-banks ($m < 0$) against a too small S-bank sector.

Part (2) of Proposition 3 is a key insight of our simple theoretical model. In an equilibrium with $m > 0$, C-bank leverage is too high relative to the planner solution. A regulator ignoring the presence of S-banks will want to increase θ. The proposition says that on the margin, this is optimal, *especially* once one accounts for the shift of intermediation activity to unregulated S-banks.

We will use our quantitative model whose parametrization we discuss in the next section to see how S-bank leverage responds to an increase in θ and to determine the optimal capital requirement. In the simple model of this section, S-bank choices are socially optimal in the sense that S-bank leverage efficiently trades off the losses from defaults against the liquidity benefit from S-bank deposits. The quantitative model takes into account that due to the lack of deposit insurance, S-bank deposits are exposed to large withdrawal shocks ("runs") that force S-banks to inefficiently liquidate assets. Further, S-banks also enjoy partial insurance of their liabilities.

4 Mapping Model To Data

4.1 Stochastic Environment And Solution Method

Stochastic processes. The stochastic process for the Y-tree (not intermediated by banks) is an AR(1) in logs

$$\log(Y_{t+1}) = (1 - \rho_Y)\log(\mu^Y) + \rho_Y \log(Y_t) + \epsilon^Y_{t+1},$$

where ϵ^Y_t is i.i.d. $\mathcal{N}$ with mean zero and volatility σ^Y. To capture the correlation of asset payoffs with fundamental income shocks, we model the productivity shock to intermediated asset as

$$Z_t = \nu^Z Y_t \exp(\epsilon_t^Z),$$

where ϵ_t^Z is i.i.d. $\mathcal{N}$ with mean zero and volatility σ^Z, independent of ϵ_t^Y, and $\nu^Z > 0$ is a parameter. This structure of the shocks implies that Z_t inherits all stochastic properties of aggregate income Y_t and is subject to a temporary shock reflecting risks specific to intermediated assets, such as credit risk. The payoff shocks ρ_t^j are *iid* and follow a Gamma distribution $\Gamma(\rho;\, \chi_0^j, \chi_1^j)$. The parameters (χ_0^j, χ_1^j) map into the mean and variance of the $F^j(\rho_t^j)$ distributions, with details in Appendix III.a.

Solution method. We solve the dynamic model using nonlinear methods. We write the equilibrium of the economy as a system of nonlinear functional equations of the state variables, with the unknown functions being the agents' choices, the asset prices, and the Lagrange multiplier on the C-bank's leverage constraint. We parametrize these functions using splines and iterate on the system until convergence. We check the relative Euler equation errors at the solution we obtain to make sure the unknown functions are well approximated. We then simulate the model for many periods and compute moments of the simulated series. For more details, refer to Appendix I.

The model features three exogenous state variables, the stochastic endowment Y_t, productivity Z_t, and the run shock π_t^R. These shocks are jointly discretized as a first-order Markov chain with three nodes for Y_t and three nodes for Z_t. We assume that runs only occur in low productivity states, yielding a total of 12 different discrete states.

The endogenous state variables are (1) the aggregate capital stock $K_t = K_t^C + K_t^S$ (recall that households do not hold any capital at the beginning of each period), the outstanding amount of bank debt of each type (2) B_t^C and (3) B_t^S, and the share of the capital stock held by S-banks (4) K_t^S / K_t. Appendix I describes the computational solution method.

4.2 Calibration

We match our model to quarterly data from 1999 Q1 to 2019 Q4 using various data sources, including bank level data from bank holding companies' (BHCs) call reports and Compustat/CRSP, as well as aggregate data from the Flow of Funds and NIPA.[24] Our calibration strategy divides parameters into two groups. The first group consists of

[24] We choose 1999 as the start date because it marked the passage of the Gramm-Leach-Bliley Act that deregulated the banking sector. For example, this legislation removed the mandated separation between commercial and investment banks. We also choose BHC data to keep the same definition of banks throughout the paper as Compustat uses BHC data.

parameters (Table 1 Panel A) that can be set in isolation to their data target. For these parameters, there is a one-to-one mapping between a parameter and a target moment in the data. Parameters of the second group (Table 1 Panel B) jointly determine different moments in our model. We choose those parameters jointly to match moments of the ergodic distribution in our model to the corresponding moments in the data.[25] That is, we start with a guess for the parameter values, solve the model with these values, then calculate the moments from the ergodic distribution, and compare them to the data. We iterate until the targeted moments in Panel B of Table 1 closely match the data. The next paragraph describes how we map key model variables to the data. Then, we discuss how we calibrate households' liquidity demand parameters. To economize on space, we defer the calibration description of all remaining parameters to Appendix Section III.b.

Data counterparts of model variables We set households' endowment income Y_t equal to real GDP per capita net of the contribution of the bank-dependent sector.[26] Bank output in the model equals the bank-dependent sector contribution to GDP, which requires us to estimate the share of bank-dependent firms. In the US, many firms are not directly dependent on banks as they can issue debt and equity in capital markets. Following the definition in Kashyap, Lamont, and Stein (1994), we classify firms as bank-dependent if they do not have a S&P long-term credit rating. Because mortgages make up the largest share of banks' loan portfolio, we also add construction and real estate firms as identified by SIC codes 6500-6599 (real estate), 1500-1599 (construction), and 1700-1799 (construction contractors, special trades) to the set of bank dependent firms. We consider all other firms as bank-independent.[27] We estimate bank-dependent GDP by applying the time series of the bank-dependent firms' sales share in Compustat to the nominal GDP series from FRED, and by deflating and dividing it by the population number.[28] On average, roughly 22% of real GDP is produced by bank dependent firms. This assumes that profit margins between bank-dependent and non-bank dependent firms are similar. We map consumption C_t to real consumption and investment I_t to real gross private domestic in-

[25]In addition to the parameters listed in Table 1, we normalize the average output of the bank-independent sector μ_Y and the average idiosyncratic shock received by banks μ^j_ρ, for $j = S, C$, to one. The latter implies that banks on average perfectly diversify away idiosyncratic shocks. We set risk aversionγ to the standard value of 2 in the macro literature (e.g., Gertler et al. (2020)).

[26]We obtain the quarterly time series of seasonally adjusted real gross-domestic product per capita, in chained 2012 dollars from FRED, Federal Reserve Bank of St. Louis. We also obtain the US population size and the GDP price index from the nominal-, real-, and per-capita GDP series.

[27]The output of bank-independent firms is captured by the endowment income Y_t.

[28]We use data from Compustat quarterly fundamentals (compm/fundq/) and Compustat's credit rating database (compm/rating/).

vestment, both downloaded from FRED, expressed in per capita terms.

In the model, C- and S-banks hold the same assets. Hence, we can choose either bank type's asset as the data counterpart for bank assets in the model. Since it is straightforward to get the asset series from regulated banks, we map assets in the model to total assets of BHCs. We deflate this series and express it in per capita terms. We map C- and S-banks' idiosyncratic payoff shocks $\rho_{i,t}^C$ and $\rho_{i,t}^B$ to equity payouts per share for individual commercial banks and shadow banks from Compustat/CRSP merged. We define shadow banks as GSE and Finance companies (27%) with SIC codes 6111-6299 (excluding SIC codes 6200, 6282, 6022, and 6199), REITS (66%) with SIC code 6798, and Miscellaneous investment firms (4%) with SIC codes 6799 and 6726. We define commercial banks as publicly traded depository institutions and bank holding companies with SIC codes from 6000 to 6089 and 6712. We compute equity payout as the quarterly dividend plus net repurchases and divide this dollar number by the shares outstanding. Using the same definition of shadow banks from above, we map S-bank leverage to the value weighted average debt to asset ratio of shadow banks in the data. The value weights are calculated using market capitalization (price times shares outstanding). We map C-bank leverage in the model to the value-weighted debt to asset ratio of BHCs.

We map C-banks' default rates to the value-weighted net-charge-off rate of BHCs' loan portfolio.[29] We map S-banks' default rates to the average default rate of bonds issued by non-traditional-banks. We get this number from the annual default study by Moody's published in January 2020.[30] We map the recovery value on C-bank debt to the recovery value on secured corporate debt from Moodys net of the resolution costs that comes from moving bank assets into FDIC receivership.[31] For shadow banks, we map their recovery value to the recovery value of unsecured debt and subordinated debt from Moodys.[32]

We set C-bank debt ($A_t^C = B_t^C$) equal to total deposits of BHCs. S-bank deposits ($A_t^S = B_t^S$) represent non-bank issued money like assets. Hence, we map them to the sum of money market mutual fund assets, Repo and commercial paper issued by non-

[29] We choose the net-loan charge off rate over bank failure rates because (i) our model abstracts from supervisory actions that allow banks to avoid failure and because (ii) both failure rates and charge-off rates are highly correlated.

[30] See exhibit 5 in `https://www.moodys.com/researchdocumentcontentpage.aspx?docid=PBC_1206734`. The non-bank financial bond default rate series has only been recently published and therefore only covers the years from 2018 to 2019.

[31] The resolution cost number is from the FDIC CFR WP 2014-04 "Understanding the Components of Bank Failure Resolution Costs".

[32] From the 2014 annual default study by Moodys, we obtain the excel file called "Default Studies - Annual Default Study Corporate Default and Recovery Rates 1920-2013 Excel data (Moodys)" from Moodys. Exhibit 8 lists the average corporate debt recovery rates by lien position from 1987-2013.

depository domestic financial institutions (using low of Fund Tables L.207, L.206, and L.209). The price on C-bank deposits q^C is mapped to the inverse of the realized interest rate on deposits. We calculate the interest rate as the value weighted ratio of aggregate interest expenses on deposits at the end of a period, divided by the total deposits at the beginning of a period. The data target for q^S is the three-month AA-rated financial commercial paper rate, obtained from FRED. Our model features a liquidity premium. To map the liquidity premium in the model to the data, we define the price $\hat{q}_t$ of a hypothetical asset that is both riskfree and void of any liquidity benefits, i.e., $\mathrm{E}_t\left[M_{t,t+1}\mathrm{MRS}_{C,t+1}\right] = q_t^C - \hat{q}_t$.[33] Unfortunately, a direct measure of $q^C - \hat{q}$ is difficult to obtain in the data since most short-term safe interest rates convey some liquidity benefit. Thus, the spread between different short-term safe rates conveys only a relative liquidity premium. Van Binsbergen, Diamond, and Grotteria (2019) propose an alternative method, by estimating a riskfree rate without a liquidity premium from option prices. The 3-month option implied riskfree rate averages 46bps per quarter from 2004 to 2018.

Liquidity demand parameters There are five parameters that determine households' liquidity demand. The parameter β governs the time discount rate and therefore scales the level of interest rates in the model. Our model determines two interest rates, one for C-banks and one for S-banks, which can be understood from the household Euler Eqs. (22) and (23) in the simple model, with their quantitative counterparts (47) and (48) in Appendix A.3. Both rates are affected by the representative consumer's stochastic discount factor, and both contain a liquidity premium. In addition, the S-bank rate reflects the default risk of S-banks. As a target for β, we use the average real interest rate BHCs pay on deposits, which over our sample period averaged 0.36% per quarter.

The parameter α is the weight on S-bank liquidity services in the CES liquidity function (19). It governs how much shadow bank debt contributes to aggregate liquidity services, and therefore affects the relative size of the S-bank sector. Thus, we target the share of shadow bank funding of real production activity, as estimated by Gallin (2015).[34]

The weight ψ on liquidity preferences scales the liquidity premium on both S-bank and C-bank debt. Using our definition of the liquidity premium in the model and the estimate of an truely risk-free asset without liquidity premium by Van Binsbergen, Diamond, and

[33] $\mathrm{MRS}_{C,t+1}$ is the marginal rate of substitution between consumption and C-bank liquidity as defined in equation (46) of Appendix A.3. The price of the hypothetical asset is $\hat{q}_t = \mathrm{E}_t\left[M_{t,t+1}\right]$.

[34] Alternatively, we could have used the share of liquid shadow bank debt (i.e., money market mutual fund shares, REPO funding, and short term commercial paper) relative to the sum of liquid shadow bank debt and bank deposits. The average share is 36% over our sample period using Flow of Funds data and therefore close to the 34% estimate by Gallin (2015).

Grotteria (2019), we calibrate ψ so that the marginal value of C-bank liquidity matches the option-rate implied liquidity premium (net of the deposit insurance fee) of 21bps.[35]

The curvature parameter γ_H and the elasticity of substitution ϵ govern the dynamic behavior of S-bank and C-bank liquidity services in our model. With $\gamma_H > 0$, the marginal value of liquidity services for either bank type decreases in the amount of aggregate liquidity provision, regardless of ϵ. That is, households have a downward-sloping demand for liquidity and γ_H governs the degree to which households respond to changes in the quantity of liquidity services. The parameter ϵ governs how much households care about the mix between S-bank and C-bank liquidity services. A value of one means S-bank and C-bank debt are perfect substitutes, a value of zero (Cobb-Douglas) means that neither or complements nor substitutes, and a value of $-\infty$ means they are perfect complements.

To infer these parameters, we run regressions in the spirit of Nagel (2016) to relate the relative prices of shadow bank and commercial bank debt to its quantity. We let the model guide us in which regression to run. We calculate the model implied spread between the prices of C- and S-bank debt. Using the FOC for C- and S-bank debt, assuming $\pi^R_{t+1} = 0$ and therefore $\xi^S = 0$, the spread is

$$q^C_t - q^S_t = \mathrm{E}_t \left[M_{t,t+1} \left(\mathrm{MRS}^C_{t+1} - \mathrm{MRS}^S_{t+1} + F^S_{\rho,t+1} \right) \right] \tag{30}$$

This allows us to write the spread in Eq. (30) as a function of the aggregate discount factor, the liquidity supply by commercial– and shadow banks, and the default rate of S-banks. We log-linearize the spread in Eq. (30) to see how γ_H and ϵ affect it. Let $M\bar{R}S \equiv \frac{MRS^C}{q^C} - \frac{MRS^S}{q^S}$ be the steady state value of the relative liquidity benefit between commercial banks and shadow banks weighted by their respective debt price. Let $\hat{x}_t$ be the deviation from the steady state. Then the log-linearized spread becomes

$$\hat{q}_{C,t} - \hat{q}_{S,t} = \mathbb{E}_t \left[\bar{m}\hat{M}_{t+1} + \bar{c}\hat{C}_{t+1} + \beta^S \hat{A}_{S,t+1} + \beta^C \hat{A}_{C,t+1} + \bar{f}\hat{F}^S_{\rho,t+1} \right], \tag{31}$$

where the coefficients $\bar{m}, \bar{c}$, and $\bar{f}$ depend on steady state values and model parameters, $\beta^j = \beta \left((1 - \epsilon - \gamma_H)\, \alpha_j M\bar{R}S \left(\frac{A_j}{H} \right)^{\epsilon} + (1 - \epsilon) \frac{MRS^j}{q^j} \right)$ for $j \in \{C, S\}$, with $\alpha_C = 1 - \alpha$ and $\alpha_S = \alpha$. This suggests a regression of the price spread $\hat{q}^C_t - \hat{q}^S_t$ on the quantity of shadow bank debt $\hat{A}_{S,t}$, and commercial bank debt $\hat{A}_{C,t}$, consumption, and proxies for the stochas-

[35] C-banks pay a deposit insurance fee κ_C for each unit of debt issued. In equilibrium, they pass on this fee to consumers in the form of lower deposit rates. Hence, we match the liquidity premium implied by the riskfree rate from Van Binsbergen, Diamond, and Grotteria (2019) and deposit rates to $\mathrm{MRS}_{C,t} - \kappa_C$ in the model.

tic discount factor and shadow banks' default rate.[36] The log-linearized spread shows that the regression coefficients β^S and β^C on shadow bank debt and commercial bank debt, respectively, are functions (among other things) of γ_H and ϵ. Note that the steady state values (e.g., $\bar{MRS}$) are also functions of γ_H and ϵ. Our calibration strategy for γ_H and ϵ targets these regression coefficients.

Table 2 presents our results.[37] In the first column, we regress the debt price spread on shadow bank and commercial bank debt (i.e., liquidity provision), controlling for consumption and the Federal Funds rate as a proxy for households' discount factor. To control for the time-varying risk of the shadow banking sector, we use the VIX index as an additional control in the second column, which is to say the risk of the market in general. This is admittedly crude, but the best we can do given the data limitations.

The parameter γ_H determines how much the marginal value of liquidity services declines when liquidity increases. This means that both q_C and q_S should be falling with the quantity of A_S. The parameter ϵ determines the relative price movement. The negative coefficient on A_S means that a higher quantity of shadow bank debt reduces q_C relatively more than q_S. Similarly, a higher quantity of A_C makes shadow bank debt relatively more expensive. This is consistent with both debt types to be net substitutes.[38] Our calibration targets the regression coefficients on A_S and A_C by running the regression in column (1) of Table 2 in our model. As a result, the calibration sets $\gamma_H = 1.6$ and $\epsilon = 0.2$.[39]

4.3 Model Fit With Data

We now turn to a discussion of the model fit. To generate the model moments in the table, we solve the model and simulate data from it. Then we calculate moments by treating the simulated data as the actual data.

In Table 1, we list the data moments that are calibration targets and compare them to the corresponding model moments. Despite its nonlinear dynamics, the model fits the

[36]We show in Appendix III.c that the log-linearized spread with $\xi^S > 0$ and therefore with the recovery rate still implies the same regression coefficients on $\hat{A}_{S,t}$ and $\hat{A}_{C,t}$.

[37]We use the HAC estimator to compute standard errors and include a constant term in the regression that is omitted from Table 2.

[38]Based on Tbill liquidity premium estimates, Nagel (2016) finds an implied elasticity of substitution between demand deposits and Tbills of one. This suggests that the elasticity of substitution between deposits and three month commercial paper is less than one, as commercial paper is less money-like than either deposits or Tbills.

[39]Including our proxy of the risk of the shadow banking sector, the estimates become more noisy but the coefficients on both debt types are similar.

targeted data moments quite well.[40] For instance, we match the volatility of commercial bank asset growth. We also match the default rates and recovery rates of shadow and commercial banks. Our model generates a slightly lower investment volatility. The model's shadow bank leverage and liquidity premium are close to data estimates.[41]

We also check the model's performance using untargeted moments listed in Table 3. Our model generates a reasonable S-bank leverage and C-bank to S-bank debt ratio volatility. The latter implies that the unconditional movement of banks' relative size is a bit smaller compared to the data. Further, it also generates reasonable business cycle correlations for most variables, that is, the correlation with GDP. We find that consumption and the investment rate are strongly procyclical in both the data and the model. Using the relative size of S-banks in liquidity provision, we also find that our model generates a positive correlation with GDP, in particular using a one-quarter lag of GDP. Our model produces similar commercial bank leverage dynamics as the data. In particular, the model rationalizes procyclical book leverage and countercyclical market leverage. The shadow bank leverage dynamics in our model are much more muted compared to the data. This is because shadow banks can maintain their target leverage ratio throughout the cycle. Using an options-implied riskfree rate (see ψ calibration discussion above for more details), the liquidity benefit in the data is countercyclical. Our model also produces a countercylical liquidity benefit. The reasons is that during an economic downturn, banks scale down their activities, leading to a lower supply of safe and liquid assets in the economy. Due to the downward sloping liquidity demand, the reduction in the supply leads to an increase in the liquidity benefit.

The model fails to produce enough C-bank leverage volatility. This is because the capital requirement constraint of commercial banks is binding in the model, whereas in the data banks hold a buffer of equity capital and thus have more flexibility to adjust leverage. Table 3 also shows that our model produces countercyclical interest rates, while in the data interest rates are procyclical. This counterfactual interest rate pattern is a known feature of models with CRRA preferences and no monetary policy shocks such as ours. See Boldrin et al. (2001) for an example that shows how preferences with habit formation can solve this issue. However, importantly for the model mechanism, the business cycle correlation of the spread between shadow bank debt and commercial bank debt has a positive sign as in the data, albeit at smaller magnitude. This means that the relative movement of rates, which matters for the competition effect, is in line with the data.

[40] Refer to Appendix III.d for details how we calculate the data series.

[41] Using a longer sample, Krishnamurthy and Vissing-Jorgensen (2012) estimate the liquidity premium to be roughly 18bps per quarter, which is very close to what our model implies.

4.4 Sensitivity Checks

Key parameters. In Table B in Appendix III.f, we conduct sensitivity checks for several parameters to verify that they indeed affect model variables as expected. The sensitivity results regarding the liquidity preference parameters ψ, α, and ϵ confirm that these parameters have distinct effects and are separately identified when calibrating the model.[42] We further demonstrate that the S-bank bailout probability π^B has a quantitatively large, nonlinear effect on S-bank leverage and defaults; the model would not be able to match observed S-bank leverage in the data with a zero probability.[43] We also analyze the sensitivity of S-bank behavior to idiosyncratic payoff risk σ_{ρ^S}. As Table B shows, this parameter determines the baseline riskiness of S-banks and its relationship to C-bank riskiness σ_{ρ^C} is quantitatively important for the response to higher capital requirements, see also the discussion in Section 5.2 below.

Liquidity Utility. As discussed in Section 2.1, the liquidity preferences in (1) assume that the relative usefulness of C- and S-bank debt for liquidity services is governed by α, the weight on S-bank debt in the CES function. The choice of $\alpha = 0.33 < 1/2$ in our calibration reflects that S-bank debt produces less liquidity services than C-bank debt per face value of debt, since it is riskier. In Appendix Table C, we explore a more general specification of the liquidity utility function that allows direct dependence of S-banks' convenience yield on S-bank default risk; in particular, we specify liquidity utility as

$$H(A_t^S, A_t^C, \mathcal{Z}_t) = \frac{\left[\Lambda(\mathcal{Z}_t)(A_t^S)^\epsilon + (1-\Lambda(\mathcal{Z}_t))(A_t^C)^\epsilon\right]^{\frac{1-\gamma_H}{\epsilon}}}{1-\gamma_H},$$

where the now time-varying weight $\Lambda(\mathcal{Z}_t)$ takes on either of two functional forms

$$\Lambda(\mathcal{Z}_t) = \alpha\left(1 - F_{\rho,t}^S\right)^\nu, \tag{A1}$$

or

$$\Lambda(\mathcal{Z}_t) = \alpha\left((1-\pi_t^R)(1 - F_{\rho,t}^S + \pi^B F_{\rho,t}^S)\right)^\nu. \tag{A2}$$

Table C shows that the quantitative effect of allowing this endogenous variation in

[42] Section 5.2 discusses the effect of changes in γ_H.

[43] Unless otherwise specified, in all quantitative results we refer to "leverage" as the conventional definition of debt over the market value of capital, $B_t^j/(p_t K_t^j)$. Note this is slightly different than the definition of leverage L^j in Section 2.3 as $B_t^j/(\Pi_t^j K_t^j)$, which we chose for expositional convenience. Quantitative differences are minor.

the relative liquidity benefit is small. There are two reasons for this small effect. First, as in the data, the average default rate and run exposure of S-banks is quantitatively minor. Second, when S-bank debt confers lower liquidity benefits at times of high S-bank defaults and during runs (as in specifications (A1) or (A2)), households substitute to C-bank debt unconditionally. The net effect is a smaller S-bank share of capital and debt in equilibrium, which is similar to solving a model with constant, but lower value of α.

4.5 Macro Effects Of Bank Runs

Bank runs make shadow banks more risky compared to commercial banks. How bad are bank runs for the economy? Figure 1 compares the impulse response functions of key model variables to a typical productivity crisis (in black) with the impulse response functions to a productivity crisis coupled with a bank run (in red). They show that a shadow bank run significantly worsens recessions, leading to higher losses in output, consumption, and investment. This is summarized by 3.5 percentage points higher deadweight losses. A bank run forces shadow banks to delever, resulting in a liquidity crunch. The lower productivity of physical capital during a run reduces the value of the intermediated assets, making investments less attractive.

5 Bank Capital Requirements

5.1 Effect Of Higher Capital Requirements

How do higher capital requirements affect aggregate liquidity provision and do they improve overall financial stability? A safer financial system is naturally the desired outcome of tighter bank regulation since the 2008 financial crisis. But what if tighter bank regulation shifts activity to the shadow banking sector? We answer this question by solving our model numerically and simulating the economy for 5,000 periods under different levels of commercial bank capital requirements. All other parameters stay at their benchmark level. The results are in Table 4.

The obvious and intended effect of higher capital requirements for C-banks is to make these banks safer by reducing their leverage. Indeed, line 6 shows that higher values of θ mechanically lower C-bank leverage. Defaults of C-banks in line 13 decline accordingly.

Do higher capital requirements make the financial system overall safer? This answer is inherently tied to the question how shadow banks react. If S-banks are more fragile, an expansion of the shadow banking sector could undo the gains in financial stability caused by tighter restrictions on C-banks' leverage. Table 4 shows that shadow banks indeed partially fill the void by providing more liquidity: the share of S-bank liabilities in overall debt (line 2) rises monotonically in the capital requirement. At a requirement of 20%, double the benchmark, the S-bank debt share is 6.9% higher. This rise in the debt share is mainly the result of higher S-bank leverage (line 5): as C-banks issue fewer deposits per dollar of assets they hold, S-bank issue more.

For moderate increases in θ, S-banks also expand their balance sheet, as both S-bank capital and the S-bank capital share in lines 3-4 increase. However, this effect reverses for large increases in the capital requirement, which cause a lower S-bank capital share.

As a result of higher leverage, S-banks become riskier: early liquidations in run episodes rise (line 7), as well as overall S-bank defaults (line 12). However, the increased S-bank default risk does not cause higher rates on S-bank debt. Instead, S-bank deposit rates decline (line 8), albeit by less than C-bank deposit rates (line 9). The decline in both rates is driven by a sharp rise in convenience yields: at the 20% capital requirement, S-banks earn a 6.3% higher convenience yield (line 10), while C-banks earn a 15.2% higher yield.

Does the increase S-bank default risk undo the gain from safer C-banks? The rise in aggregate consumption (line 16) shows that this is not the case. Consumption rises monotonically with higher capital requirements since deadweight losses caused by C-bank defaults fall. Further, the demand effect (see Section 3.3) causes greater capital accumulation (line 1) and aggregate output (line 15) increases. The resource gains from these two channels more than offset the greater losses from higher S-bank defaults. The gain in consumption further dominates the loss in liquidity production (line 15), leading to increased welfare (line 19) relative to the benchmark for capital requirements up to 30%.[44] The optimal capital requirement that maximizes aggregate welfare is at 16%.

5.2 Understanding The Effects

We unpack the model response to an increase in the capital requirement by considering three simplified versions of our quantitative model in Table 5. These numerical experiments allow us to relate our quantitative results to the analytical insights from Section 3.3. With our calibrated model at hand, we can now study the quantitative strength of the demand effect and competition effect discussed in Section 3.3. To hone in on the competition effect, we consider three model variations that turn the *demand effect off* by setting

[44] We measure welfare in terms of the value function of households.

$\gamma_H = 0$ in the liquidity function (19).[45] By comparing Model (3) to the benchmark model discussed in the previous section, we can then understand the role of the demand effect.

We begin with Model (1) in Table 5 that only features C-banks. In this model variation, C-banks own all intermediated assets and only their deposits enter households' liquidity utility. Model (1) delivers a simple trade-off for capital requirements: higher θ lowers C-bank leverage (line 5) and defaults (line 12), thereby raising consumption (line 15), but reducing liquidity services (line 14). Since C-banks defaults are close to completely eliminated at a capital requirement around 15%, further increases do not improve welfare.

In Model (2), we add S-banks, but impose several simplifications relative to the full model: (i) S-banks are not subject to runs, (ii) the probability that their debt will be bailed out is zero, and (iii) the three parameters governing default and recovery, which are the dispersion of bank-idiosyncratic shocks σ_{ρ^S}, the default penalty δ_S, and the recovery fraction ξ^S are the same as for C-banks. We label Model (2) "Simple Model", since it is closest in assumptions to the two-period model in Section 3. Raising capital requirements in Model (2) reduces C-bank defaults and increases consumption like in Model (1). However, since S-banks hold on average 41.8% of intermediated capital, the reduction in C-bank defaults and bankruptcy losses per unit of assets causes a smaller rise in consumption (line 15) in Model (2) relative to Model (1). The key differences to Model (1) is the *competition effect* discussed in Section 3.2. Raising θ reduces the profitability of C-bank equity relative to S-bank equity. Thus, the S-bank share increases both for debt and capital (lines 2 and 3). This improves the mix of liquidity services that is distorted away from the optimum at a low capital requirement. Hence, liquidity declines less rapidly with increases in θ (line 14). Welfare is maximized at 15% for Model (1) and 14% for Model (2).

The benefit of tighter capital regulation is smaller in Model (2), with a welfare increase of 0.0349% compared to 0.0709% in Model (1). It is important to stress that this does not result from leakage to riskier S-banks. Rather, S-banks in Model (2) are close to perfectly safe with a baseline default rate of 0.01%. The benefit from regulating C-banks is simply smaller since they make up a smaller share of the intermediation sector to begin with.

Absent the demand effect, the simple model (see Corollary (1) in Section 3.3) predicts that S-bank leverage should decline for an increase in the capital requirement. This prediction is also borne out by the dynamic version of the simple model. Due to the competition effect, the reduction in C-bank leverage lowers the competitive pressures that drive up S-bank leverage, leading to a reduction in S-bank leverage and risk-taking.

[45] Since γ_H affects both the scale and the elasticity of the liquidity premium, we recalibrate ψ for each of the experiments to keep the scale unchanged.

In Model (3), we add back S-bank runs, probabilistic bailouts, and S-bank sector specific parameters (σ_{ρ^S}, δ_S, ξ^S). The only difference between Model (3) and the full quantitative model in Table 4 is the absence of demand effects ($\gamma_H = 0$ in Model (3)). The main differences between Models (2) and (3) are due to S-banks now being "runnable". As a result, they have a higher baseline default rate of 0.33% and a smaller market share of 33.41%. As before, raising the capital requirement causes C-banks to become safer and consumption to rise. As in Model (2) the competition effect causes a rise in the S-bank share (lines 2-3), and S-banks reduce leverage (line 4), also becoming safer (line 11). Since the fragile S-bank calibration in Model (3) implies that S-banks contribute meaningfully to deadweight losses, a reduction in S-bank default also contributes to the rise in consumption. As a result, consumption rises by more than in Model (2). The welfare gains from higher θ are greater than in Model (2), but smaller than in Model (1). The main take-away from Model (3) is that, due to the competition effect, the presence of riskier S-banks does not reduce the benefits of higher capital requirements on C-banks, despite shifting market share to S-banks. Tighter regulation of C-banks lower the distortionary advantages these banks enjoy, which in turn allows S-banks to reduce risk-taking in equilibrium.

Going from Model (3) in Table 5 to the full quantitative model in Table 4 adds the demand effect by setting $\gamma_H = 1.6$. Comparing the results for both models, the main consequence of the demand effect is that the convenience yields of both banks rise strongly with higher θ in the full model (lines 10-11 in Table 4), whereas without the demand effect the S-bank convenience yield declines and the C-bank yield rises but less strongly (lines 9-10 in Table 5). This differential response of yields has profound consequences for the equilibrium cost of capital: without demand effect, higher θ causes the economy to shrink, leading to a smaller capital stock and GDP (lines 1 and 14 of Table 5). With demand effect, the economy expands (lines 1 and 15 of Table 4). Furthermore, the demand effect dominates the competition effect, causing S-banks to increase leverage (line 6) and thus leading to more defaults (line 12).

To sum up, in the counterfactual world without a demand effect (Table 5, Model (3)) higher capital requirements cause the economy to shrink, but S-banks become safer. With a demand effect as in Table 4, the same increase in θ causes the economy to expand, but S-banks become riskier. Quantitatively, the demand effect interacts positively with higher capital requirements: the expansion of GDP and consumption outweigh the greater risk-taking of S-banks. In our model with a demand effect, welfare is maximized at a 16% capital requirement with a gain of 0.054% relative to the benchmark. If there was no demand effect (Model (3)), the optimal capital requirement would be lower at 15%, and the welfare gain would be smaller at 0.047%.

5.3 Shadow Banking Pre- And Post-Financial Crisis

We did not design the quantitative model to explain trends in the size of the shadow banking sector in the U.S. financial system. However, the financial crisis of 2008 set in motion large changes in financial regulation that can be viewed through the lens of our model. Implementation of Basel III capital regulation, different measures of the Dodd-Frank act, and stress testing for large banks post-crisis effectively lead to moderate increases in capital requirements for banks.[46] Further, Berndt, Duffie, and Zhu (2019) have argued that after the crisis, the willingness of regulators and government agencies to rescue off-balance sheet operations and other shadow banking vehicles declined substantially. We view the pre-crisis period as one of relatively lax capital requirements, with large implicit guarantees for shadow banks. We further follow Moreira and Savov (2017) in assuming that agents were underestimating the risk of a run on shadow banks.

To capture these changes in our model, we start the simulation in Q2 of 2008 with a pre-crisis parametrization that makes the following changes relative to the benchmark calibration in Section 4: (i) a lower capital requirement at $\theta = 8\%$, (ii) a higher S-bank bailout probability $\pi^B = 87\%$, and (iii) zero (perceived) probability of a run. Then, reflecting the collapse of Lehman Brothers and the ensuing distress in money markets, the economy experiences a run on shadow banks and a bad productivity shock. Following that, changes in financial regulation over three years lead to an increase in the capital requirement to 11% and a decline in the bailout probability for S-banks to 70%. Furthermore, agents now correctly anticipate the possibility of future S-bank runs. During this period, both exogenous shocks follow their stochastic laws of motion.

The solid black lines with circles in Figure 2 plot this scenario. The dotted blue line shows a counterfactual scenario, in which capital requirements remain at the pre-crisis level of 8%, isolating their effect. A run episode in our model triggers a sharp contraction in output of the bank-dependent sector (top row, left panel), see also Section 4.5. Consumption drops by close to 1% and liquidity services decline by over 15% (middle and right panels). The bottom row compares the response of the financial system to the run shock and subsequent regulatory changes to the data, see Appendix III.e for a description of the data series. The initial share of S-banks in debt markets is high at 42%, close to the data, due to the high bailout probability and underestimation of run risk. During the run, shadow banks' market share declines sharply, and then settles at a new lower level as households update on run risk and regulators lower the bailout probability. S-bank leverage follows a similar trajectory, matching high pre-crisis and low post-crisis leverage of

[46] See e.g., Greenwood et al. (2017); Duffie (2018); Tarullo (2019).

shadow banks in the data.

By comparing the solid black and dotted blue lines, we can see the effect of only raising capital requirements. C-bank leverage (bottom row) directly responds to the tightening of capital requirements, exhibiting the same decline as in the data. With tighter capital requirements, consumption recovers to a higher and liquidity production to a lower level than pre-crisis. Consistent with the results in Table 4, the increase in θ leads to a higher S-bank share by roughly 1 pp (bottom left). However, since investors realize the correct run risk and S-bank bailouts are less likely, the S-bank share still drops by 10 pp.

This simulation shows that our model can capture many aspects of the post-crisis changes to the financial system. While the model supports the narrative that tighter regulation post-crisis caused an expansion in shadow banking, this effect is quantitatively dwarfed by other changes that led to a decline in shadow banking.[47]

6 Conclusion

We propose a quantitative general equilibrium framework that views unregulated shadow banks as alternative providers of credit and liquidity services to analyze the unintended consequences of capital requirements on regulated banks. Our model highlights and quantifies two opposing general equilibrium effects that together determine how shadow banks respond to tighter regulation of commercial banks.

Our analysis shows that tighter regulation leads indeed to substitution towards shadow banking. However, it also clarifies that a financial system with more shadow banking is not necessarily riskier. In our calibration, tighter regulation of commercial banks leads to larger and riskier shadow banks, yet increases the stability of the financial system overall.

While the focus on funding differences of traditional and shadow banks sharpens our conclusions, our framework abstracts from differences in technology or expertise on the asset side. Studying these differences in our framework would be a fruitful avenue for future research. Further, we only indirectly account for the government's role in safe asset provision in that the government is the ultimate backstop for failing banks. However, in the data the government also provides liquidity directly. Interactions of government debt supply with shadow banking is an important question for future work.

[47]Notice that our definition of shadow banking is based on short-term liabilities in money markets: we consider as shadow banks those financial institutions that fund illiquid assets such as loans by issuing short-term runnable debt. Using this definition, we see a decline in shadow banking post-crisis. Other definitions of shadow banks, such as non-bank mortgage originators that sell mortgages to the GSEs as in e.g. Buchak et al. (2018), yield different post-crisis trends in shadow banking.

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Table 1: Calibration

Panel A: Parameters with a one-to-one mapping to the data

	Value	Description	Target
		Production	
ρ_Y	0.599	Non-bank GDP autocorr.	AC(NIPA GDP - Bank dep. GDP)
σ^Y	0.87%	Non-bank GDP volatility	Vol(NIPA GDP - Bank GDP)
δ_K	2.5%	Depreciation	Quarterly depreciation NIPA
η	0.667	Labor share	NIPA labor share
σ_{ρ^C}	12.1%	SD C-bank idiosync. shocks	vol(equity payout C-banks)
σ_{ρ^S}	25.4%	SD S-bank idiosync. shocks	vol(equity payout S-banks)
		Bank regulation	
θ	10%	Capital requirement	mean(Tier-1 Eq / Assets)
κ	0.142%	Deposit insurance fee	FDIC report (see table notes)
		Runs	
$\underline{Z}$	$26\% \times Z$	Real asset foreclosure discount	Campbell et al. (2011)
π^R	[0, 0.333]	Fraction of households run	Covitz et al. (2013)
Prob_{π^R}	$\begin{bmatrix} 0.963 & 0.0375 \\ 0.75 & 0.25 \end{bmatrix}$	Unconditional run probability Average bank run length	4.76% 1.3 quarters

Panel B: Parameters that jointly match moments in the data

Values		Description	Target	Data	Model
			Bank technology		
ν^Z	0.23	Scales bank output	Real GDP per capita share	22.3%	22.3%
σ^Z	1.74%	Bank dep. output volatility	Vol(Bank output growth)	2.93%	3.02%
ϕ_I	0.3	Investment adj. cost	Vol(Investment)	2.65%	1.04%
ϕ_K	0.011	Capital growth adj. cost	Vol(C-bank asset growth)	0.50%	0.46%
			Bank default		
δ_S	0.39	Default penality S-banks	S-bank bond default bond default rate	0.28%	0.30%
δ_C	0.204	Default penality C-banks	Loan net charge-offs	0.23%	0.23%
ξ_S	20.5%	Bankruptcy cost S-banks	Unsecured debt recov. Moody's	38.1%	38.2%
ξ_C	35.2%	Bankruptcy cost C-banks	Sec. debt recov. & FDIC resol. costs	48.1%	48.1%
π_B	85%	Bailout probability	Shadow bank leverage	87.0%	83.2%
			Liquidity preferences		
β	0.993	Discount rate	C-bank interest rate on deposits	0.36%	0.39%
α	0.33	CES weight S-bank debt	S-bank share Gallin (2015)	34.0%	33.7%
ψ	0.0072	Liq. preference weight	Liq. premium BDG2019	0.21%	0.17%
γ_H	1.6	Liq. preference curvature	Reg. coefficient on AS	-0.19%	-0.14%
ϵ	0.2	Liq. type elasticity	Reg. coefficient on AC	0.50%	0.68%
			Run		
$\underline{\delta}_K$	2.5%	capital dep.	Avg. haircut Gorton and Metrick (2009)	15.1%	15.2%

Notes: This table lists the parameters of our model. See discussion in Section 4.2 on data counterparts of model variables and liquidity demand parameters. All other parameters are discussed in Appendix III.b. The data sources for this table are from NIPA, Flow of Funds, FRED (`https://fred.stlouisfed.org/`), Compustat/CRSP and the FR-Y-9C. Sample period is from 1999Q1 to 2019Q4. The link to the FDIC report is `https://www.fdic.gov/about/strategic/report/2016annualreport/ar16section3.pdf`. The volatility of equity payout is the time series average from Compustat/CRSP of the cross-sectional volatility of equity payout (dividend + net repurchases) per share for commercial banks and shadow banks (see appendix for definition using SIC codes). Using the same definition and data source, shadow bank leverage is the value weighted debt to asset ratio. BDG2019 stands for Van Binsbergen, Diamond, and Grotteria (2019).

Table 2: Commercial − Shadow bank debt price spread regressions

	(1)	(2)
log(S-bank debt/GDP)	-0.19%	-0.14%
	(-1.86)	(-1.13)
log(C-bank debt/GDP)	0.50%	0.44%
	(1.64)	(1.45)
Federal Funds Rate	54.89%	54.61%
	(10.02)	(10.66)
log(C/GDP)	0.57%	-0.24%
	(0.26)	(-0.11)
VIX		-0.12%
		(-1.39)
adj. R2	0.77	0.78

Notes: The sample is 1999Q1 – 2019Q4. The dependent variable is the quarterly spread between the price on commercial bank debt, defined as the inverse of the interest rate paid on deposits, and the price on shadow bank debt, defined as the 3-month AA-financial commercial paper price. The interest rate paid on deposits is calculated as the aggregate amount of interest rate expense on deposits divided by the aggregate amount of deposits. The data comes from FR-Y-9C reports (bank holding companies). We download the 3-month AA-financial commercial paper rate from FRED (i.e., Federal Reserve Economic Data maintained by the Federal Reserve bank of St. Louis). We download the Fed Funds rate from FRED. Shadow bank debt is the sum of REPO claims held by shadow banks (Flow of Funds table L.207), total money market mutual fund assets (equals liabilities) (table L.206), and commercial paper held by the domestic financial sector less depository institutions (table L.209). Commercial bank debt is set to aggregate bank deposits (FR-Y-9C). We download the real consumption series from FRED. All quantities are normalized by GDP and logged. The VIX is the daily CBOE Volatility Index downloaded from FRED, converted into a quarterly average. We use the HAC estimator to compute heteroscedasticity and autocorrelation consistent standard errors. We report the resulting t-statistics in brackets. Both regressions include a constant term.

Table 3: Properties of the dynamic model

	Data	Model
Volatility		
Consumption	0.38%	0.39%
Mkt Lev C	1.48%	0.10%
Book Lev C	0.29%	0.09%
Mkt Lev S	2.22%	3.51%
Book Lev S	3.41%	3.51%
Debt C/Debt S	0.52	0.20
Business Cycle Correlation		
Consumption	0.89	0.72
Investment	0.92	0.95
Debt S Share	0.43	0.03
Debt S (lag GDP)	0.26	0.15
Book Lev C	0.16	0.43
Mkt Lev C	-0.21	-0.42
Book Lev S	0.04	-0.00
Mkt Lev S	-0.31	-0.02
C-bank liquidity benefit	-0.26	-0.52
yield C	0.74	-0.85
yield S	0.82	-0.79
rf. rate	0.70	-0.85
spread S-C	0.51	0.12

This table presents untargeted moments in the model and compares them to their data counterpart. The sample for the data is 1999 Q1 to 2019 Q4. The volatility is the standard deviation of the HP-filtered variable. The GDP correlation is the correlation of HP-filtered business cycle component of GDP with the HP-filtered business cycle component of the variable of interest. See Appendix III.d for the description of the data.

Table 4: Effect of higher capital requirements

	Base	13%	14%	15%	16%	17%	20%	30%
Capital and Debt								
1. Capital	3.15	0.16%	0.23%	0.30%	0.38%	0.46%	0.72%	1.64%
2. Debt share S	31.95%	2.71%	3.39%	4.01%	4.61%	5.18%	6.91%	13.79%
3. Capital share S	33.68%	0.26%	0.09%	-0.15%	-0.43%	-0.73%	-1.73%	-4.79%
4. Capital S	1.06	0.42%	0.32%	0.16%	-0.05%	-0.28%	-1.02%	-3.23%
5. Leverage S	83.18%	0.18%	0.26%	0.34%	0.43%	0.52%	0.80%	1.80%
6. Leverage C	89.95%	-3.33%	-4.44%	-5.56%	-6.67%	-7.78%	-11.12%	-22.22%
7. Early Liquidation (runs)	0.00	0.25%	0.36%	0.47%	0.59%	0.72%	1.10%	2.51%
Prices								
8. Deposit rate S	0.45 %	-0.66%	-0.96%	-1.28%	-1.61%	-1.96%	-3.05%	-6.80%
9. Deposit rate C	0.39%	-3.69%	-4.86%	-6.01%	-7.17%	-8.35%	-12.04%	-26.83%
10. Convenience Yield S	0.28%	1.39%	2.00%	2.65%	3.33%	4.04%	6.26%	14.33%
11. Convenience Yield C	0.31%	4.68%	6.14%	7.60%	9.06%	10.54%	15.17%	33.97%
Welfare								
12. Default S	0.30%	3.05%	4.40%	5.85%	7.39%	9.00%	14.12%	34.08%
13. Default C	0.23%	-65.11%	-76.16%	-83.96%	-89.38%	-93.09%	-98.28%	-100.00%
14. GDP	1.29	0.01%	0.02%	0.02%	0.03%	0.03%	0.05%	0.12%
15. Liquidity Services	1.48	-2.16%	-2.85%	-3.54%	-4.22%	-4.90%	-6.96%	-14.09%
16. Consumption	1.21	0.062%	0.073%	0.081%	0.086%	0.090%	0.098%	0.107%
17. Vol(Liquidity Services)	0.03	-3.16%	-4.26%	-5.39%	-6.53%	-7.66%	-11.08%	-22.31%
18. Vol(Consumption)	0.00	0.31%	0.34%	0.36%	0.36%	0.36%	0.31%	0.07%
19. Welfare		0.0460%	0.0511%	0.0535%	0.0540%	0.0527%	0.0435%	0.0053%

This tables shows the moments of the simulated model for different values of C-banks' capital requirement θ. The "Benchmark" titled column shows the moments for the benchmark calibration. All other columns show the percentage change relative to the benchmark moment.

Table 5: Higher capital requirements without demand effect ($\gamma_H = 0$)

	(1) C-bank Only				**(2) Simple Model**				**(3) No Demand Effect**			
	B	**14**	**15**	**16%**	**B**	**14**	**15**	**16%**	**B**	**14**	**15**	**16%**
	Capital and Debt											
1. Capital	3.17	-0.64%	-0.76%	-0.89%	3.20	-0.38%	-0.45%	-0.52%	3.17	-0.40%	-0.47%	-0.54%
2. Debt share S	-	-	-	-	39.73%	4.37%	5.24%	6.08%	31.75%	4.15%	4.95%	5.72%
3. Capital share S	-	-	-	-	41.83%	1.60%	1.77%	1.89%	33.41%	1.29%	1.34%	1.33%
4. Leverage S	-	-	-	-	82.47%	-0.10%	-0.12%	-0.15%	83.47%	-0.42%	-0.50%	-0.58%
5. Leverage C	89.99%	-4.44%	-5.55%	-6.66%	89.99%	-4.45%	-5.56%	-6.67%	89.94%	-4.43%	-5.54%	-6.65%
6. Early Liquidation (runs)	-	-	-	-	-	-	-	-	0.00	-0.62%	-0.73%	-0.84%
	Prices											
7. Deposit rate S	-	-	-	-	0.46%	1.92%	2.30%	2.65%	0.43%	1.69%	2.01%	2.31%
8. Deposit rate C	0.37%	0.09%	0.14%	0.22%	0.35%	-1.93%	-2.31%	-2.68%	0.38%	-1.34%	-1.60%	-1.83%
9. Convenience Yield S	-	-	-	-	0.24%	-3.76%	-4.50%	-5.21%	0.29%	-3.15%	-3.75%	-4.32%
10. Convenience Yield C	0.33%	0.20%	0.22%	0.23%	0.35%	1.94%	2.32%	2.68%	0.32%	1.58%	1.88%	2.15%
	Welfare											
11. Default S	-	-	-	-	0.01%	-4.01%	-4.81%	-5.57%	0.33%	-6.62%	-7.87%	-9.05%
12. Default C	0.23%	-76.20%	-83.97%	-89.37%	0.23%	-76.26%	-84.02%	-89.42%	0.23%	-76.14%	-83.95%	-89.38%
13. GDP	1.29	-0.05%	-0.06%	-0.07%	1.29	-0.03%	-0.03%	-0.04%	1.29	-0.03%	-0.04%	-0.04%
14. Liquidity Services	2.85	-5.06%	-6.28%	-7.50%	1.47	-3.77%	-4.64%	-5.50%	1.49	-3.72%	-4.60%	-5.48%
15. Consumption	1.21	0.10%	0.11%	0.12%	1.21	0.06%	0.07%	0.07%	1.21	0.07%	0.07%	0.08%
16. Vol(Liquidity Services)	0.06	-3.98%	-5.02%	-6.04%	0.03	-3.76%	-4.65%	-5.52%	0.05	0.43%	0.30%	0.12%
17. Vol(Consumption)	0.00	0.31%	0.27%	0.19%	0.00	0.61%	0.69%	0.76%	0.00	0.00%	0.02%	0.04%
18. Welfare		0.070%	0.071%	0.069%		0.035%	0.034%	0.031%		0.046%	0.047%	0.045%

This tables presents moments of the simulated model for different model versions and different values of C-banks' capital requirements θ. All cases are calculated for $\gamma_H = 0$, which means that there is no demand effect.

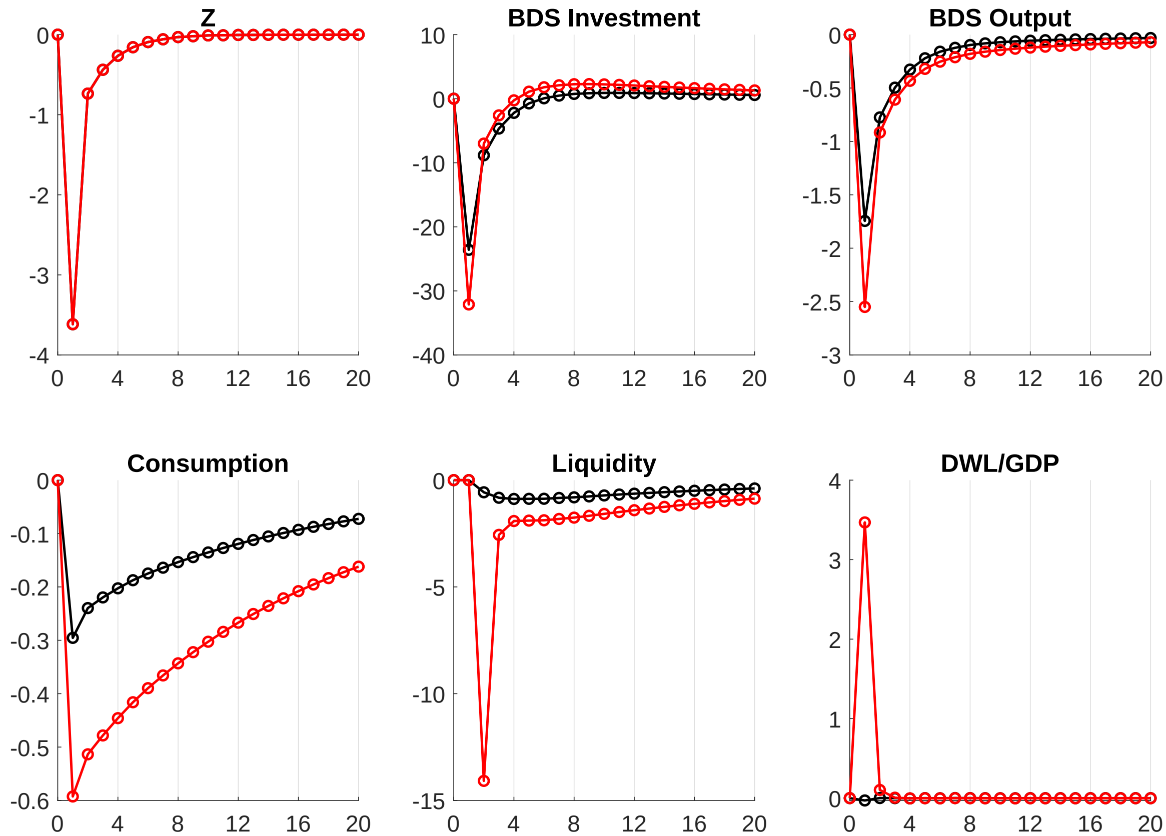

Figure 1: The effect of bank runs on the economy

This figure presents the impulse response functions to a productivity shock (in black) and a productivity shock together with a run shock (red). The x-axis denotes quarters. The shocks occur in the first quarter. The y-axis denotes percentage deviations from the stationary equilibrium for all plots but bottom right (DWL/GDP), which shows the difference to the stationary equilibrium in percentage points.

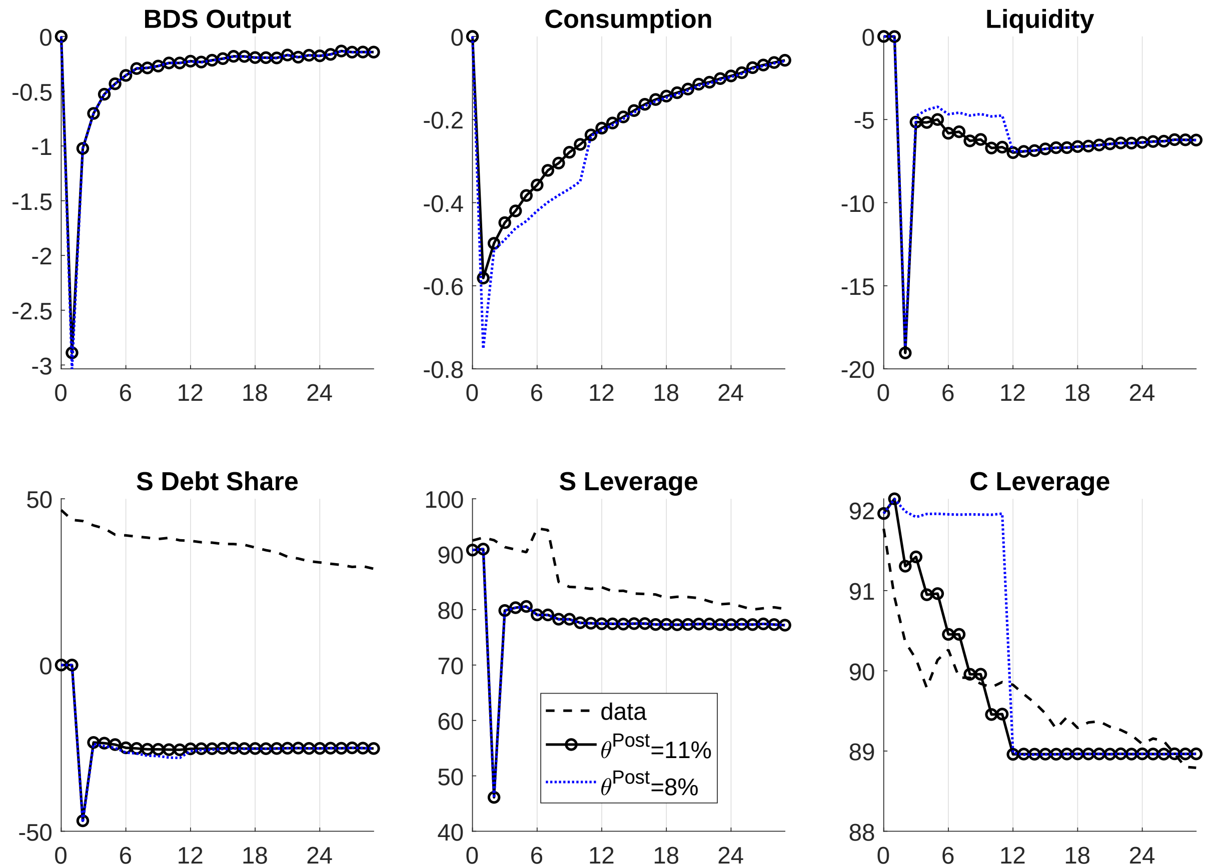

Figure 2: Recovery from 2008 financial crisis in model simulations

This figure plots the time path of several model variables in a simulation of the 2008 financial crisis. The y-axis denotes percentage deviations from the initial state for the four panels in the top row, and percent in the four bottom panels. The solid black lines with circles plot the baseline simulation described in the text that raises capital requirements to 11% post-crisis. The dotted blue line includes the same parameter changes as the black line, except the increase in capital requirements. For the bottom three panels, the dashed line plots data counterparts to the model variables as described in Appendix III.e.

A Quantitative Model Appendix

This section describes the quantitative model in detail. Time is discrete and infinite. Households receive stochastic endowment Y_t from a Lucas tree. When the run shock realizes, S-banks need to sell a fraction of their assets, i.e., physical capital, within the period to households who have a lower valuation. Production executed by banks is exposed to an aggregate shock Z_t, and production executed by households is exposed to an aggregate shock $\bar{Z}_t$. Bank dividends are subject to idiosyncratic shocks ρ_t^j.

We introduce two types of adjustment costs, investment- and balance sheet adjustment costs.

A.1 Bank Optimization and Aggregation

This section describes details of the bank optimization problem resulting in equations (7) and (9) in the main text. Anticipating the result that all banks are equal due to i.i.d. shocks and a value function that is homogeneous in capital, we suppress individual bank subscripts throughout.

Production. After aggregate productivity Z_t and the run shock π_t^R are realized, all banks produce and invest. Denote by $\hat{K}_t^j = (1-\ell_t^j)K_t^j$ the capital banks retain after a possible fire sale due to runs. The profits generated by the two lines of bank business (production and investment) are

$$\hat{D}_t^j = Z_t \left(\hat{K}_t^j\right)^{1-\eta}\left(N_t^j\right)^{\eta} + (1-\delta_K)p_t\hat{K}_t^j - w_t N_t^j + I_t^j\left(p_t - 1\right) - \frac{\phi_I}{2}\left(\frac{I_t^j}{\hat{K}_t^j} - \delta_K\right)^2 \hat{K}_t^j, \qquad (32)$$

for $j = S, C$. Banks choose labor input N_t^j and investment I_t^j to maximize (32). Note that the profit also includes the proceeds from selling depreciated capital after production, $(1-\delta_K)p_t\hat{K}_t^j$.

The first-order condition for labor demand is the usual intratemporal condition equating the wage to the marginal product of labor

$$w_t = Z_t\eta\left(\frac{N_t^j}{\hat{K}_t^j}\right)^{\eta-1} = Z_t\eta\left(n_t^j\right)^{\eta-1}. \qquad (33)$$

Similarly, the first-order condition for investment yields the usual relationship between the capital price and the marginal value of a unit of capital

$$p_t = 1 + \phi_I\left(\frac{I_t^j}{\hat{K}_t^j} - \delta_K\right) = 1 + \phi_I\left(i_t^j - \delta_K\right). \qquad (34)$$

We can substitute both conditions back into the definition of profit in (32) to eliminate the wage and investment and define the gross payoff per unit of capital in equation (2) to get

$$\hat{D}_t^j = \Pi_t^j \hat{K}_t^j.$$

The total dividend banks pay to shareholders is given by

$$D_t^j = \rho_t^j \hat{D}_t^j - (1 - \pi_t^R I_{j=S}) B_t^j + (q_t^j - \kappa_j) B_{t+1}^j - p_t K_{t+1}^j - \frac{\phi_K}{2} \left(\frac{K_{t+1}^j}{\hat{K}_t^j} - 1 \right)^2 \hat{K}_t^j.$$

It scales the profit banks receive from their real business, $\hat{D}_t^j$, by the idiosyncratic shock, ρ_t^j, and also includes redemptions of last period's deposits, B_t^j, and the equity cost of the portfolio for next period, $(q_t^j - \kappa_j) B_{t+1}^j - p_t K_{t+1}^j - \frac{\phi_K}{2} \left(\frac{K_{t+1}^j}{\hat{K}_t^j} - 1 \right)^2 \hat{K}_t^j$, where the deposit insurance fee $\kappa_j = 0$ for S-banks in the benchmark model.

Bank value function. We define the value function of a bank that did not default, at the time it chooses its portfolio for next period as

$$\hat{V}^j(\hat{K}_t^j, \rho_t^j, \mathcal{Z}_t) \quad = \quad \max_{K_{t+1}^j, B_{t+1}^j} D_t^j \; + \; \mathrm{E}_t \left[M_{t,t+1} \max \left\{ \hat{V}^j(\hat{K}_{t+1}^j, \rho_{t+1}^j, \mathcal{Z}_{t+1}) \, , -\delta_j \Pi_{t+1}^j \hat{K}_{t+1}^j \right\} \right]. \quad (35)$$

We assume that the default penalty $-\delta_j \Pi_{t+1}^j K_{t+1}^j$ in (35) is proportional to the asset value of the bank with parameter δ_j. This is reasonable and also retains the homogeneity of the problem in capital $\hat{K}_t^j$.

To simplify the problem, we recognize that profits from real business and deposits redemptions $\rho_t^j \hat{D}_t^j - (1 - \pi_t^R I_{j=S}) B_t^j$ are irrelevant for the bank's choice after the default decision. After the bankruptcy decision, non-bankrupt banks choose their portfolio for next period, and households set up new banks to replace those banks who defaulted. With respect to the portfolio choice for period $t+1$, the optimization problem of all banks is identical conditional on having the same capital $\hat{K}_t^j$. Thus we define the value function

$$V^j(\hat{K}_t^j, \mathcal{Z}_t) = \hat{V}^j(\hat{K}_t^j, \rho_t^j, \mathcal{Z}_t) - \rho_t^j \hat{D}_t^j + (1 - \pi_t^R I_{j=S}) B_t^j,$$

such that we can write the problem in (35) equivalently as

$$\begin{aligned} V^j(\hat{K}_t^j, \mathcal{Z}_t) = & \max_{K_{t+1}^j, B_{t+1}^j} (q_t^j - \kappa_j) B_{t+1}^j - p_t K_{t+1}^j - \frac{\phi_K}{2} \left(\frac{K_{t+1}^j}{\hat{K}_t^j} - 1 \right)^2 \hat{K}_t^j \\ & + \mathrm{E}_t \left[M_{t,t+1} \max \left\{ \rho_{t+1}^j \Pi_{t+1}^j \hat{K}_{t+1}^j - B_{t+1}^j (1 - \pi_{t+1}^R I_{j=S}) + V^j(\hat{K}_{t+1}^j, \mathcal{Z}_{t+1}) \, , -\delta_S \Pi_{t+1}^j \hat{K}_{t+1}^j \right\} \right], \quad (36) \end{aligned}$$

where runs lower banks outstanding liabilities.

Aggregation. Next, using the notation for the dependance on the aggregate state vector $\mathcal{Z}_t$, we conjecture that $V_t^j(\hat{K}_t^j)$ is homogeneous in capital $\hat{K}_t^j$ of degree one. This allows us to define the scaled value function $v_t^j = \frac{V_t^j(\hat{K}_t^j)}{\hat{K}_t^j}$. Defining the fire sale discount as $x_t^j = \frac{\Pi_t^H}{\Pi_t^j}$, capital structure as $b_{t+1}^j \equiv \frac{B_{t+1}^j}{K_{t+1}^j}$, leverage as $L_{t+1}^j \equiv \frac{b_{t+1}^j}{\Pi_{t+1}^j}$, asset growth $k_{t+1}^j \equiv \frac{K_{t+1}^j}{\hat{K}_t^j}$, and notice that $\frac{\hat{K}_t^j}{K_t^j} = 1 - \ell_t^j$, we can write $v^j(Z)$ as

$$v_t^j = \max_{k_{t+1}^j, b_{t+1}^j} \left((q_t^j - \kappa_j) b_{t+1}^j - p_t \right) k_{t+1}^j - \frac{\phi_K}{2} \left(k_{t+1}^j - 1 \right)^2$$
$$+ k_{t+1}^j \mathrm{E}_t \left[M_{t,t+1} \Pi_{t+1}^j \max \left\{ \left(1 - \ell_{t+1}^j\right) \left(\rho_{t+1}^j + \frac{v_t^j}{\Pi_{t+1}^j} \right) - (1 - \pi_{t+1}^R I_{j=S}) L_{t+1}^j \, , -\delta_j \left(1 - \ell_{t+1}^j\right) \right\} \right]. \tag{37}$$

Generalizing the definition of the default threshold $\hat{\rho}_t^j$ in the main text in equation (8) for both banks, we define the leverage-adjusted payoff of banks' portfolio including the default option

$$\Omega_t^j(L_t^j) \equiv \max \left\{ \left(1 - \ell_{t+1}^j\right) \left(\rho_{t+1}^j + \frac{v_t^j}{\Pi_{t+1}^j} \right) - (1 - \pi_{t+1}^R I_{j=S}) L_{t+1}^j \, , -\delta_j \left(1 - \ell_{t+1}^j\right) \right\}. \tag{38}$$

Note that taking the expectation with respect to ρ_{t+1}^j allows us to rewrite the max operator in equation (37) such that

$$\mathrm{E}_{\rho^j} \left[\Omega_t^j(L_t^j) \right] = (1 - F_{\rho,t}^j) \left(\left(1 - \ell_t^j\right) \left(\rho_t^{j,+} + \frac{v_t^j}{\Pi_t^j} \right) - (1 - \pi_{t+1}^R I_{j=S}) L_t^j \right) - F_{\rho,t}^j \delta_j \left(1 - \ell_{t+1}^j\right),$$

where $F_{\rho,t}^j = F^j(\hat{\rho}_t^j)$ is the probability of default and $\rho_t^{j,+} = \mathrm{E}\left(\rho_t^j \mid \rho_t^j > \hat{\rho}_t^j\right)$ is the expected value of the idiosyncratic shock conditional on not defaulting.

We can thus rewrite (37) more compactly as

$$v_t^j = \max_{k_{t+1}^j, b_{t+1}^j} \left((q_t^j - \kappa_j) b_{t+1}^j - p_t \right) k_{t+1}^j - \frac{\phi_K}{2} \left(k_{t+1}^j - 1 \right)^2$$
$$+ k_{t+1}^j \mathrm{E}_t \left[M_{t,t+1} \Pi_{t+1}^j \Omega_{t+1}^j (L_{t+1}^j) \right]. \tag{39}$$

Equation (39) corresponds to equations (7) and (9) in the main text. It shows that the expectation only depends on the capital structure choice through leverage $L_{t+1}^j = b_{t+1}^j / \Pi_{t+1}^j$. The first-order condition for asset growth k_{t+1}^j is

$$p_t - (q_t^j - \kappa_j) b_{t+1}^j + \phi_K \left(k_{t+1}^j - 1 \right) = \mathrm{E}_t \left[M_{t,t+1} \Pi_{t+1}^j \Omega_{t+1}^j (L_{t+1}^j) \right]. \tag{40}$$

Substituting (40) into (39) yields

$$v_t^j = k_{t+1}^j \phi_K (k_{t+1}^j - 1) - \frac{\phi_K}{2} \left(k_{t+1}^j - 1 \right)^2 = \frac{\phi_K}{2} \left((k_{t+1}^j)^2 - 1 \right).$$

The solution for v_t^j confirms the conjecture that

$$V_t^j(\hat{K}_t^j) = \hat{K}_t^j v_t^S,$$

and we can thus solve the problem of a representative bank of each type. Note that the scaled value function v_t^j only depends on asset growth k_{t+1}^j, which is a choice variable. Intuitively, if the bank expects high future capital growth, then the continuation value is large.

A.2 Equilibrium Definition

Given a sequence of aggregate $\{Y_t, Z_t, \pi_t^R\}$ and idiosyncratic shocks $\{\rho_{t,i}^S, \rho_{t,i}^C\}$, a competitive equilibrium consists of a sequence of prices $\{w_t, p_t, q_t^S, q_t^C, p_t^S, p_t^C\}$, household choices $\{C_t, A_{t+1}^S, A_{t+1}^C, S_t^S, S_t^C, N_t^H\}$, S-bank choices $\{I_t^S, N_t^S, B_{t+1}^S, K_{t+1}^S\}$, and C-bank choices $\{I_t^C, N_t^C, B_{t+1}^C, K_{t+1}^C\}$ such that households and banks optimize given prices, and markets clear.

There is market clearing for capital

$$K_{t+1}^S + K_{t+1}^C = I_t^S + I_t^C + (1-\delta_K) \sum_{j=S,C} \left(1 - \xi^j F_{\rho,t}^j \rho_t^{j,-}\right) K_t^j \left(1 - \ell_t^j\right) + (1 - \underline{\delta}_K) K_t^S \ell_t^S, \tag{41}$$

securities issued by banks

$$\begin{aligned} B_{t+1}^S &= A_{t+1}^S, \\ B_{t+1}^C &= A_{t+1}^C, \\ S_t^S &= 1, \\ S_t^C &= 1, \end{aligned}$$

the goods market

$$\begin{aligned} C_t + \sum_{j=S,C} \left(I_t^j + \Phi(I_t^j, K_t^j)\right) + \sum_{j=S,C} \mathrm{DWL}_t^j & \\ = Y_t + Z_t \sum_{j=S,C} (N_t^j)^\eta \left((1-\ell_t^j)K_t^j\right)^{1-\eta} &+ \underline{Z}_t (N_t^H)^\eta (\ell_t^S K_t^S)^{1-\eta}, \end{aligned} \tag{42}$$

and the labor market

$$N_t^S + N_t^C + N_t^H = 1. \tag{43}$$

The deadweight losses for each bank type are

$$\mathrm{DWL}_t^j = \xi^j F_{\rho,t}^j \rho_t^{j,-} \left(1 - \ell_t^j\right) \left(\Pi_t^j - (1-\delta_K) p_t\right) K_t^j,$$

and the adjustment costs for capital and investment amount to

$$\Phi(I_t^j, K_t^j) = K_t^j \left(1 - \ell_t^j\right) \left(\frac{\phi_I}{2} \left(i_t^j - \delta_K\right)^2 + \frac{\phi_K}{2} \left(k_t^j - 1\right)^2\right).$$

Note that commercial banks are isolated from bank runs, and so $\ell_t^C = 0 \; \forall t$. The market clearing condition for capital in Equation (41) is also the transition law for the aggregate capital stock. Bank failures lead to additional depreciation endogenously determined by the failure rate of banks $F_{\rho,t}^j$: when a bank defaults, a fraction of $\xi^j \rho_t^j$ of that bank's capital is destroyed. At the same time, condition (42) shows that bank failures also lead to a loss of production, such that fewer resources are available in the goods market.

A.3 Household Problem

Denoting household wealth at the beginning of the period by W_t, the complete intertemporal problem of households is

$$V_t^H(A_t^S, A_t^C, W_t) = \max_{C_t, A_{t+1}^S, A_{t+1}^C, S_t^S, S_t^C} U(C_t, H\left(A_t^S, A_t^C\right)) + \beta\,\mathrm{E}_t\left[V_{t+1}(A_{t+1}^S, A_{t+1}^C, W_{t+1})\right]$$

subject to the budget constraint in (12). The transition law for household financial wealth W_t is

$$\begin{aligned} W_{t+1} = & \sum_{j=S,C} (1 - F_{\rho,t+1}^j)\left(D_{t+1}^{j,+} + p_{t+1}^j\right) S_t^j \\ & + \left(1 - \pi_{t+1}^R\right) A_{t+1}^S \left[1 - F_{\rho,t+1}^S + F_{\rho,t+1}^S \left(\pi_B + (1-\pi_B)\, r_{t+1}^S\right)\right] + \pi_{t+1}^R A_{t+1}^S \\ & + A_{t+1}^C, \end{aligned}$$

which clarifies that S-bank liquidity services are risky. The beginning-of-period dividend paid by S-banks to households conditional on survival is

$$D_t^{S,+} = \rho_t^{S,+} \hat{K}_t^S \Pi_t^S - (1-\pi_t^R) B_t^S + \hat{K}_t^S \left(k_{t+1}^S \left(q_t^S b_{t+1}^S - p_t\right) - \frac{\phi_K}{2}\left(k_{t+1}^S - 1\right)^2\right),$$

where $\hat{K}_t^S = \left(1 - \ell_t^S\right) K_t^S$, and for C-banks dividends are

$$D_t^{C,+} = \rho_t^{C,+} \hat{K}_t^C\, \Pi_t^C - B_t^C + \hat{K}_t^C \left(k_{t+1}^C \left((q_t^C - \kappa) b_{t+1}^C - p_t\right) - \frac{\phi_K}{2}\left(k_{t+1}^C - 1\right)^2\right),$$

where $\hat{K}_t^C = K_t^C$ since $\ell_t^C = 0$. Households' first-order conditions for purchases of bank equity are, for $j = S, C$,

$$p_t^j = \mathrm{E}_t\left[M_{t,t+1}\left(1 - F_{\rho,t+1}^j\right)\left(D_{t+1}^{j,+} + p_{t+1}^j\right)\right], \tag{44}$$

where we have defined the stochastic discount factor

$$M_{t,t+1} = \beta\,\frac{U_C(C_{t+1}, H_{t+1})}{U_C(C_t, H_t)} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

The marginal rate of substitution between consumption and liquidity services of bank type j is defined as

$$\mathrm{MRS}_{j,t} = \frac{U_H(C_t, H_t)}{U_C(C_t, H_t)} \frac{\partial H(A_t^S, A_t^C)}{\partial A_t^j},$$

and given by

$$\mathrm{MRS}_{S,t} = \alpha \psi C_t^{\gamma} H_t^{-\gamma_H} \left(\frac{H_t}{A_t^S}\right)^{1-\epsilon}, \tag{45}$$

$$\mathrm{MRS}_{C,t} = (1-\alpha) \psi C_t^{\gamma} H_t^{-\gamma_H} \left(\frac{H_t}{A_t^C}\right)^{1-\epsilon}. \tag{46}$$

for S- and C-bank debt, respectively.

Then the first-order conditions for purchases of S-bank debt and C-bank debt are:

$$q_t^S = \mathrm{E}_t \left\{ M_{t,t+1} \left[(1-\pi_{t+1}^R) \left(1 - F_{\rho,t+1}^S + F_{\rho,t+1}^S(\pi_B + (1-\pi_B) r_{t+1}^S) \right) + \pi_{t+1}^R + \mathrm{MRS}_{S,t+1} \right] \right\}. \tag{47}$$

$$q_t^C = \mathrm{E}_t \left\{ M_{t,t+1} \left[1 + \mathrm{MRS}_{C,t+1} \right] \right\}. \tag{48}$$

The payoff of commercial bank debt is 1, whereas the payoff of shadow bank debt depends on their default probability, recovery value, and the probability of a government bailout π_B. The last term in each expectation represents the marginal benefit of liquidity services to households, as defined in (45) and (46).

A.4 S-bank Optimality Conditions

Each period, S-banks choose investment I_t^S, labor input N_t^S, capital growth k_{t+1}^S and capital structure b_{t+1}^S. The first-order conditions for investment and labor are given by (34) and (33), respectively. They are incorporated into the gross payoff of capital Π_t^S in (2). The first-order condition for capital growth is given by (40).

It remains to derive the first-order condition for capital structure. Before doing so, we first recognize that individual S-banks take into account the effect of their capital structure choice the price of their debt. Thus, they optimally respond to households' valuation of idiosyncratic S-bank risk, such that we replace q_t^S in (39) by the function

$$q_S(b_{t+1}^S) = \mathrm{E}_t \left\{ M_{t,t+1} \left[(1-\pi_{t+1}^R) \left(1 - F_{\rho,t+1}^S + F_{\rho,t+1}^S(\pi_B + (1-\pi_B) r_{t+1}^S) \right) + \pi_{t+1}^R + \mathrm{MRS}_{S,t+1} \right] \right\},$$

which is households' first-order condition for S-bank debt purchases in (47). Differentiating equation (39) with respect to b_{t+1}^S after this substitution yields

$$q_t^S + b_{t+1}^S \, q_S'(b_{t+1}^S) = \mathrm{E}_t \left[M_{t+1} \Pi_{t+1}^S \frac{\partial L_{t+1}^S}{\partial b_{t+1}^S} \frac{\partial \Omega_{t+1}^S(L_{t+1}^S)}{\partial L_{t+1}^S} \right].$$

To compute the partial derivatives on the right-hand side note that

$$\frac{\partial L_{t+1}^S}{\partial b_{t+1}^S} = \frac{1}{\Pi_{t+1}^S}$$

and that

$$\frac{\partial \Omega_{t+1}^S(L_{t+1}^S)}{\partial L_{t+1}^S} = -I_{[\rho_{t+1}^S > \hat{\rho}_{t+1}^S]} \left(1 - \pi_{t+1}^R + \frac{\ell_{t+1}^S}{L_{t+1}^S} \left(\rho_{t+1}^S + \frac{v_{t+1}^S}{\Pi_{t+1}^S} \right) \right),$$

which in turn uses the derivative

$$\frac{\partial (1-\ell_t^S)}{\partial L_t^S} = -\frac{\ell_t^S}{L_t^S}.$$

Inserting these expression and taking expectations with respect to the distribution of ρ^S_{t+1} yields

$$q^S_t + b^S_{t+1}\, q'_S(b^S_{t+1}) = \mathrm{E}_t\left[M_{t+1}(1-F^S_{t+1})\left(1-\pi^R_{t+1}+\frac{\ell^S_{t+1}}{L^S_{t+1}}\left(\rho^{S,+}_{t+1}+\frac{v^S_{t+1}}{\Pi^S_{t+1}}\right)\right)\right]. \tag{49}$$

To obtain the partial derivative $q'_S(b^S_{t+1})$, we differentiate equation (47) to get

$$q'_S(b^S_{t+1}) = (1-\pi_B)\mathrm{E}_t\left[(1-\pi^R_{t+1})M_{t+1}\frac{\partial L^S_{t+1}}{\partial b^S_{t+1}}\left(\frac{\partial F^S_{t+1}r^S_{t+1}}{\partial L^S_{t+1}}-\frac{\partial F^S_{t+1}}{\partial L^S_{t+1}}\right)\right].$$

In section A.4.1 we calculate $\frac{\partial F^S_{t+1}r^S_{t+1}}{\partial L^S_{t+1}}-\frac{\partial F^S_{t+1}}{\partial L^S_{t+1}}$, such that the derivative becomes

$$\begin{aligned}q'_S(b^S_{t+1}) = &-\frac{1-\pi_B}{b^S_{t+1}}\mathrm{E}_t\Bigg\{M_{t,t+1}\Bigg[\frac{(1-\xi^S)F^S_{t+1}\rho^{S,-}_{t+1}}{L^S_{t+1}}\\ &+f^S_{t+1}\mathcal{L}^S(L^S_{t+1})\left((1-\xi_S)(1-\ell^S_{t+1})\left(\delta_S+\frac{v^S_{t+1}}{\Pi^S_{t+1}}\right)+\xi_S(1-\pi^R_{t+1})L^S_{t+1}\right)\Bigg]\Bigg\},\end{aligned} \tag{50}$$

where

$$\mathcal{L}^S(L^S_t) = \frac{\partial\hat{\rho}^S_t}{\partial L^S_t}$$

is the derivative of the default threshold with respect to leverage (equation (51)).

The full first-order condition for the S-bank capital structure choice is obtained by substituting (50) into (49).

It is useful to examine the FOC for the case of no run, $\pi^R_{t+1} = 0$ and $\ell^S_{t+1} = 0$, zero default penalty $\delta_S = 0$, no capital adjustment cost $\phi_K = 0$ (implying $v^S_{t+1} = 0$), and zero bailout probability $\pi_B = 0$. In that case, the derivative in (50) reduces to

$$q'_S(b^S_{t+1}) = -\frac{1}{b^S_{t+1}}\mathrm{E}_t\left\{M_{t,t+1}\left[F^S_{t+1}r^S_{t+1}+\xi_S f^S_{t+1}L^S_{t+1}\right]\right\}$$

and the first-order condition (49) becomes

$$q^S_t + b^S_{t+1}\, q'_S(b^S_{t+1}) = \mathrm{E}_t\left[M_{t+1}(1-F^S_{t+1})\right].$$

Combining these two equations, we get

$$q^S_t = \mathrm{E}_t\left[M_{t+1}\left(1-F^S_{t+1}+F^S_{t+1}r^S_{t+1}+\xi_S f^S_{t+1}L^S_{t+1}\right)\right].$$

We can equate this with the household first-order condition (47) and collect terms to get

$$\mathrm{E}_t\left[M_{t+1}\xi_S f^S_{t+1}L^S_{t+1}\right] = \mathrm{E}_t\left[M_{t+1}\mathrm{MRS}_{S,t+1}\right].$$

This equation is the analogue to equation (10) in the simple model. The S-bank chooses leverage to equalize the expected marginal liquidity benefit to households on the RHS with the expected marginal losses due to bankruptcy on the LHS.

A.4.1 Computing $q'_S(b^S_{t+1})$

Computing $\frac{\partial F^S_{t+1} r^S_{t+1}}{\partial L^S_{t+1}}$. Recall the definition of the recovery value for S-banks as

$$r^S(L^S_t) = (1-\xi^S)\frac{\rho^{S,-}_t(1-\ell^S_t)}{(1-\pi^R_t)L^S_t},$$

with the conditional expectation $\rho^{S,-}_t = \mathrm{E}\left[\rho \,|\, \rho < \hat{\rho}^S_t\right]$.

We can rewrite the recovery value times the probability of default as

$$F^S_t r^S_t = \frac{1-\xi_S}{(1-\pi^R_t)L^S_t}(1-\ell^S_t)\int_{-\infty}^{\hat{\rho}^S_t} \rho \, dF^S(\rho).$$

First, we compute the derivative of the default threshold with respect to L^S_t as

$$\mathcal{L}^S_t(L^S_t) = \frac{\partial \hat{\rho}^S_t}{\partial L^S_t} = \frac{1-\pi^R_t}{(1-\ell^S_t)^2}. \tag{51}$$

Then differentiating $F^S_t r^S_t$ with respect to L^S_t gives

$$\begin{aligned}
\frac{\partial F^S_t r^S_t}{\partial L^S_t} =& -\frac{1-\xi_S}{(1-\pi^R_t)(L^S_t)^2}(1-\ell^S_t)F^S_t\rho^{S,-}_t \\
&+ \frac{1-\xi_S}{(1-\pi^R_t)L^S_t}\left[-\frac{\ell^S_t}{L^S_t}F^S_t\rho^{S,-}_t + f^S_t\mathcal{L}^S(L^S_t)\left((1-\pi^R_t)L^S_t - (1-\ell^S_t)\left(\delta_S - \frac{v^S_t}{\Pi^S_t}\right)\right)\right] \\
=& -\frac{1-\xi_S}{(1-\pi^R_t)L^S_t}\left[\frac{F^S_t\rho^{S,-}_t}{L^S_t} + f^S_t\mathcal{L}^S(L^S_t)\left((1-\ell^S_t)(\delta_S + \frac{v^S_t}{\Pi^S_t}) - (1-\pi^R_t)L^S_t\right)\right],
\end{aligned}$$

where we use the function $\mathcal{L}^S(L^S_t)$ defined in (51).

Combining. Using that

$$\frac{\partial F^S_{t+1}}{\partial L^S_{t+1}} = f^S_t\mathcal{L}^S(L^S_t),$$

we get

$$\begin{aligned}
&\frac{\partial F^S_{t+1} r^S_{t+1}}{\partial L^S_{t+1}} - \frac{\partial F^S_{t+1}}{\partial L^S_{t+1}} \\
&= -\frac{(1-\xi_S)F^S_t\rho^{S,-}_t}{(1-\pi^R_t)(L^S_t)^2} - \frac{f^S_t}{(1-\pi^R_t)L^S_t}\mathcal{L}^S(L^S_t)\left((1-\xi_S)(1-\ell^S_t)\left(\delta_S + \frac{v^S_t}{\Pi^S_t}\right) + \xi_S(1-\pi^R_t)L^S_t\right).
\end{aligned}$$

A.5 C-bank Optimality Conditions

Like S-banks, C-banks choose investment I_t^C, labor input N_t^C, capital growth k_{t+1}^C and capital structure b_{t+1}^C. The first-order conditions for investment and labor are given by (34) and (33), respectively. They are incorporated into the gross payoff of capital Π_t^C in (2). The first-order condition for capital growth is given by (40).

To derive the C-bank first-order condition for capital structure, first note that C-banks are subject to the regulatory constraint in (10). Denote the Lagrange multiplier associated with the constraint by λ_t^C. Differentiating (39) subject to the constraint with respect to b_{t+1}^C gives

$$q_t^C - \kappa_C = \lambda_t^C - \mathrm{E}_t \left[M_{t+1} \Pi_{t+1}^C \frac{\partial L_{t+1}^C}{\partial b_{t+1}^C} \frac{\partial \Omega_{t+1}(L_{t+1}^C)}{\partial L_{t+1}^C} \right],$$

where based on (38)

$$\Omega_t(L_t^C) = \max \left\{ \rho_{t+1}^C + \frac{v_t^C}{\Pi_{t+1}^C} - L_{t+1}^C \, , -\delta_C \right\}.$$

The partial derivatives on the right-hand side are

$$\frac{\partial L_{t+1}^C}{\partial b_{t+1}^C} = \frac{1}{\Pi_{t+1}^C}$$

and

$$\frac{\partial \Omega_{t+1}(L_{t+1}^C)}{\partial L_{t+1}^C} = -I_{[\rho_{t+1}^C > \hat{\rho}_{t+1}^C]}.$$

Therefore, the first-order condition, after taking expectations with respect to the distribution of ρ_{t+1}^C, is

$$q_t^C - \kappa_C = \lambda_t^C + \mathrm{E}_t \left[M_{t+1}(1 - F_{t+1}^C) \right]. \tag{52}$$