Financial Regulation in a Quantitative Model of the Modern Banking System: Online Appendix

I Computational Solution Method

The equilibrium of dynamic stochastic general equilibrium models is usually characterized recursively. If a stationary Markov equilibrium exists, there is a minimal set of state variables that summarizes the economy at any given point in time. Equilibrium can then be characterized using two types of functions: transition functions map today's state into probability distributions of tomorrow's state, and policy functions determine agents' decisions and prices given the current state. Brumm et al. (2018) analyze theoretical existence properties in this class of models and discuss the literature. Perturbation-based solution methods find local approximations to these functions around the "deterministic steady-state". For applications in finance, there are often several problems with local solution methods. First, portfolio restrictions such as leverage constraints may be occasionally binding in the true stochastic equilibrium. Generally, a local approximation around the steady state (with a binding or slack constraint) will therefore inaccurately capture nonlinear dynamics when constraints go from slack to binding. Further, local methods have difficulties in dealing with highly nonlinear functions within the model such as probability distributions or option-like payoffs, as is the case for the quantitative model in this paper. Finally, in models with rarely occurring bad shocks (such as the runs in our model), the steady state used by local methods may not properly capture the ergodic distribution of the true dynamic equilibrium.

Global projection methods (Judd (1998)) avoid these problems by not relying on the deterministic steady state. Rather, they directly approximate the transition and policy functions in the relevant area of the state space.

I.a Equilibrium Conditions

The solution of the model can be written as a system of 17 nonlinear functional equations in equally many unknown functions of the state variables. The model's state variables are $\mathcal{S}_t = (Y_t, Z_t, \pi^R_t, K^C_t, K^S_t, A^C_t, A^S_t)$.

The functions are aggregate consumption $C(\mathcal{S}_t)$, prices of C-bank and S-bank equity

$(p^C(\mathcal{S}_t), p^S(\mathcal{S}_t))$, prices of C-bank and S-bank deposits $(q^C(\mathcal{S}_t), q^S(\mathcal{S}_t))$, C-bank and S-bank deposit issuance per unit of capital $(b^C(\mathcal{S}_{t+1}), b^S(\mathcal{S}_{t+1}))$, the Lagrange multiplier on C-bank leverage $\lambda^C(\mathcal{S}_t)$, C-bank and S-bank capital purchases $(K^C(\mathcal{S}_{t+1}), K^S(\mathcal{S}_{t+1}))$, the capital price $p(\mathcal{S}_t)$, C-bank and S-bank investment $(I^C(\mathcal{S}_t), I^S(\mathcal{S}_t))$, labor demand of C-bank, S-bank and households $(N^C(\mathcal{S}_t), N^S(\mathcal{S}_t), N^H(\mathcal{S}_t))$, and the wage $w(\mathcal{S}_t)$. For the equations, we will use time subscripts und suppress the dependence on state variables. All variables can be expressed as functions of current ($\mathcal{S}_t$) or one-period ahead ($\mathcal{S}_{t+1}$) state variables.

The equations are

$$p_t^C = \mathrm{E}_t\left[M_{t,t+1}F^C_{\rho,t+1}\left(D^{C,+}_{t+1} + p^C_{t+1}\right)\right] \tag{E1}$$

$$p_t^S = \mathrm{E}_t\left[M_{t,t+1}F^S_{\rho,t+1}\left(D^{S,+}_{t+1} + p^S_{t+1}\right)\right] \tag{E2}$$

$$q_t^C = \mathrm{E}_t\left[M_{t,t+1}\left(1 + \mathrm{MRS}^C_{t+1}\right)\right] \tag{E3}$$

$$q_t^S = \mathrm{E}_t\left[M_{t,t+1}\left((1-\pi^R_{t+1})\left(1 - F^S_{\rho,t+1}\left(1 - (\pi_B + (1-\pi_B)r^S_{t+1})\right)\right) + \pi^R_{t+1} + \mathrm{MRS}^S_{t+1}\right)\right] \tag{E4}$$

$$\begin{aligned} C_t + I_t^C + I_t^S + \Phi(I_t^C, K_t^C) + \Phi(I_t^S, (1-\ell_t^S)K_t^S) &= Y_t + Y_t^C + Y_t^S + Y_t^H \\ &- \xi^C F_t^C \rho_t^{C,-}(\Pi_t^C - (1-\delta_K)p_t)K_t^C \\ &- \xi^S F_t^S \rho_t^{S,-}(1-\ell_t^S)(\Pi_t^S - (1-\delta_K)p_t)K_t^S \end{aligned} \tag{E5}$$

$$q_t^S + b^S_{t+1}\, q_S'(b^S_{t+1}) = \mathrm{E}_t\left[M_{t+1}(1 - F^S_{\rho,t+1})\left(1 - \pi^R_{t+1} + \frac{\ell_{t+1}}{L^S_{t+1}}\left(\rho^{S,+}_{t+1} + \frac{v^S_{t+1}}{\Pi^S_{t+1}}\right)\right)\right] \tag{E6}$$

$$q_t^C - \kappa = \lambda_t^C + \mathrm{E}_t\left[M_{t,t+1}(1 - F^C_{\rho,t+1})\right] \tag{E7}$$

$$\lambda_t^C\left(p_t - (1-\theta)b^C_{t+1}\right) = 0 \tag{E8}$$

$$p_t - q_t^S b^S_{t+1} + \phi_K\left(k^S_{t+1} - 1\right) = \mathrm{E}_t\left[M_{t,t+1}\Pi^S_{t+1}\,\Omega^S\left(L^S_{t+1}\right)\right], \tag{E9}$$

$$p_t - (q_t^C - \kappa)b^C_{t+1} + \phi_K\left(k^C_{t+1} - 1\right) = \mathrm{E}_t\left[M_{t,t+1}\Pi^C_{t+1}\,\Omega^C\left(L^C_{t+1}\right)\right] \tag{E10}$$

$$\begin{aligned} K^C_{t+1} + K^S_{t+1} &= I_t^C + I_t^S + (1-\delta_K)\left(1 - \xi^C F^C_{\rho,t}\rho_t^{C,-}\right)K_t^C \\ &+ (1-\delta_K)\left(1 - \xi^S F^S_{\rho,t}\rho_t^{S,-}\right)(1-\ell_t^S)K_t^S + (1-\underline{\delta}_K)\ell_t^S K_t^S \end{aligned} \tag{E11}$$

$$I_t^C = \left(\frac{p_t - 1}{\phi_I} + \delta_K\right)K_t^C \tag{E12}$$

$$I_t^S = \left(\frac{p_t - 1}{\phi_I} + \delta_K\right)(1-\ell_t^S)K_t^S \tag{E13}$$

$$w_t = \eta Z_t (n_t^C)^{\eta - 1} \tag{E14}$$

$$N_t^C = \frac{K_t^C}{K_t^C + (1-\ell_t^S)K_t^S + \ell_t^S K_t^S(\underline{Z}_t/Z_t)^{1/(1-\eta)}} \tag{E15}$$

$$N_t^S = \frac{(1-\ell_t^S)K_t^S}{K_t^C}N_t^C \tag{E16}$$

$$N_t^H = 1 - N_t^C - N_t^S \tag{E17}$$

(E1) – (E4) are the household Euler equations for bank equity and debt from equations (44) applied to $j = C, S$, (47), and (48). (E5) is the resource constraint from (42). (E6) is the

S-bank condition for leverage from (49). (E7) is the C-bank condition for leverage (52), with (E8) being the complementary slackness condition for the leverage constraint (10). (E9) and (E10) are the S-bank and C-bank conditions for capital growth from (40), applied to either bank type. (E11) is the market clearing condition for capital (41), and (E12) – (E13) are the first-order conditions for investment by banks from (34), applied to $j = C, S$. (E14) – (E16) are the first-order conditions for labor demand by banks and households, from (33) applied to $j = C, S, H$, and (E17) is the market clearing condition for labor.

I.b Solution Procedure

The projection-based solution approach used in this paper has three main steps.

Step 1. **Define approximating basis for the policy and transition functions.** To approximate these unknown functions, we discretize the state space and use multivariate linear interpolation. Our general solution framework provides an object-oriented MATLAB library that allows approximation of arbitrary multivariate functions using linear interpolation, splines, or polynomials. For the model in this paper, splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation.

Step 2. **Iteratively solve for the unknown functions.** Given an initial guess for policy and transition functions, at each point in the discretized state space compute the current-period optimal policies. Using the solutions, compute the next iterate of the transition functions. Repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose. This step is completely parallelized across points in the state space within each iterate.

Step 3. **Simulate the model for many periods using approximated functions.** Verify that the simulated time path stays within the bounds of the state space for which policy and transition functions were computed. Calculate relative Euler equation errors to assess the computational accuracy of the solution. If the simulated time path leaves the state space boundaries or errors are too large, the solution procedure may have to be repeated with optimized grid bounds or positioning of grid points.

We will now provide a more detailed description for each step.

Step 1 The state space consists of

- two exogenous state variables $[Y_t, \pi_t^R]$, and
- four endogenous state variables $[K_t, K_t^S, B_t^S, B_t^C]$.

The banking sector specific shock Z_t does not contain any persistent shocks in addition to Y_t and is therefore not an additional state variable. We first discretize Y_t into a N^Y-state Markov chain using the Rouwenhorst (1995) method, where N^Y is an odd number. The procedure chooses the productivity grid points $\{Y_j\}_{j=1}^{N^Y}$ and the $N^Y \times N^Y$ Markov transition matrix Π_Y between them to match the volatility and persistence of GDP growth of the bank independent sector. The run shock π_t^R can take on two realizations $\{0, \bar{\pi}^R\}$ as described in the calibration section. The 2 x 2 Markov transition matrix between these states is given by Π_{π^R}. We assume that run shocks only occur in states with negative GDP growth. Denote the set of the $N^x = N^Y + (N^Y - 1)/2$ values the exogenous state variables can take on as $\mathcal{S}_x$, and the associated Markov transition matrix Π_x.

Our solution algorithm requires approximation of continuous functions of the endogenous state variables. Define the "true" endogenous state space of the model as follows: if each endogenous state variable $S_t \in \{K_t, K_t^S, B_t^S, B_t^C\}$ can take on values in a continuous and convex subset of the reals, characterized by constant state boundaries, $[\bar{S}_l, \bar{S}_u]$, then the endogenous state space $\mathcal{S}_n = [\bar{K}_l, \bar{K}_u] \times [\bar{K}_l^S, \bar{K}_u^S] \times [\bar{B}_l^S, \bar{B}_u^S] \times [\bar{B}_l^C, \bar{B}_u^C]$. The total state space is the set $\mathcal{S} = \mathcal{S}_x \times \mathcal{S}_n$.

To approximate any function $f : \mathcal{S} \to \mathcal{R}$, we form an univariate grid of (not necessarily equidistant) strictly increasing points for each endogenous state variables, i.e., we choose $\{K_j\}_{j=1}^{N_K}$, $\{K_k^S\}_{k=1}^{N_{K^S}}$, $\{B_m^S\}_{m=1}^{N_{B^S}}$, and $\{B_n^C\}_{n=1}^{N_{B^C}}$. These grid points are chosen to ensure that each grid covers the ergodic distribution of the economy in its dimension, and to minimize computational errors, with more details on the choice provided below. Denote the set of all endogenous-state grid points as $\hat{\mathcal{S}}_n = \{K_j\}_{j=1}^{N_K} \times \{K_k^S\}_{k=1}^{N_{K^S}} \times \{B_m^S\}_{m=1}^{N_{B^S}} \times \{B_n^C\}_{n=1}^{N_{B^C}}$, and the total discretized state space as $\hat{\mathcal{S}} = \mathcal{S}_x \times \hat{\mathcal{S}}_n$. This discretized state space has $N^S = N^x \cdot N_K \cdot N_{K^S} \cdot N_{B^S} \cdot N_{B^C}$ total points, where each point is a 5 x 1 vector as there are 5 distinct state variables (counting the exogenous state as one). We can now approximate the smooth function f if we know its values $\{f_j\}_{j=1}^{N^S}$ at each point $\hat{s} \in \hat{S}$, i.e. $f_j = f(\hat{s}_j)$ by multivariate linear interpolation.

Our solution method requires approximation of of three sets of functions defined on the domain of the state variables. The first set of unknown functions $\mathcal{C}_P : \mathcal{S} \to \mathcal{P} \subseteq \mathcal{R}^{N^C}$, with N^C being the number of policy variables, determines the values of endogenous objects specified in the equilibrium definition at every point in the state space. These are the prices, agents' choice variables, and the Lagrange multipliers on the portfolio constraints. Specifically, the 8 policy functions are debt prices $q^S(\mathcal{S})$, $q^C(\mathcal{S})$, capital price

$p(\mathcal{S})$, debt issued by banks in the current period $B^S(\mathcal{S})$, $B^C(\mathcal{S})$, the capital purchased by S-banks $K^S(\mathcal{S})$, labor demand of S-banks $n^S(\mathcal{S})$, and the Lagrange multiplier for the C-bank leverage constraint $\lambda^C(\mathcal{S})$. There is an equal number of these unknown functions and nonlinear functional equations, to be listed under step 2 below.

The second set of functions $\mathcal{C}_T : \mathcal{S} \times \mathcal{S}_x \to \mathcal{S}_n$ determine the next-period endogenous state variable realizations as a function of the state in the current period and the next-period realization of exogenous shocks. There is one transition function for each endogenous state variable, corresponding to the transition law for each state variable, also to be listed below in step 2.

The third set are forecasting functions $\mathcal{C}_F : \mathcal{S} \to \mathcal{F} \subseteq \mathcal{R}^{N^F}$, where N^F is the number of forecasting variables. They map the state into the set of variables sufficient to compute expectations terms in the nonlinear functional equations that characterize equilibrium. They partially coincide with the policy functions. In particular, the forecasting functions for our model are the capital price $p(\mathcal{S})$, S-bank labor input $n^S(\mathcal{S})$, capital growth of both types of banks $k^S(\mathcal{S})$, $k^C(\mathcal{S})$, and the value function of households $V^H(\mathcal{S})$ (to compute welfare).

Step 2 Given an initial guess $\mathcal{C}^0 = \{\mathcal{C}_P^0, \mathcal{C}_T^0, \mathcal{C}_F^0\}$, the algorithm to compute the equilibrium takes the following steps.

A. **Initialize** the algorithm by setting the current iterate $\mathcal{C}^m = \{\mathcal{C}_P^m, \mathcal{C}_T^m, \mathcal{C}_F^m\} = \{\mathcal{C}_P^0, \mathcal{C}_T^0, \mathcal{C}_F^0\}$.

B. **Compute forecasting values.** For each point in the discretized state space, $s_j \in \hat{\mathcal{S}}$, $j = 1, \ldots, N^S$, perform the steps:

   i. Evaluate the transition functions at s_j combined with each possible realization of the exogenous shocks $x_i \in \mathcal{S}_x$ to get $s_j^{'}(x_i) = \mathcal{C}_T^m(s_j, x_i)$ for $i = 1, \ldots, N^x$, which are the values of the endogenous state variables given the current state s_j and for each possible future realization of the exogenous state.

   ii. Evaluate the forecasting functions at these future state variable realizations to get $f_{i,j}^0 = \mathcal{C}_F^m\left(s_j^{'}(x_i), x_i\right)$.

   The end result is a $N^x \times N^S$ matrix $\mathscr{F}^m$, with each entry being a vector

$$f_{i,j}^m = [p_{i,j}, n_{i,j}^S, k_{i,j}^S, k_{i,j}^C, V_{i,j}^H] \tag{F}$$

   of the next-period realization of the forecasting functions for current state s_j and future exogenous state x_i.

C. **Solve system of nonlinear equations.** At each point in the discretized state space, $s_j \in \hat{\mathcal{S}}$, $j = 1, \ldots, N^S$, solve the system of nonlinear equations that characterize equilibrium in the equally many "policy" variables, given the forecasting matrix $\mathscr{F}^m$ from step B. This amounts to solving a system of 12 equations in 12 unknowns

$$\hat{P}_j = [\hat{q}_j^S, \hat{q}_j^C, \hat{p}_j, \hat{B}_j^S, \hat{B}_j^C, \hat{K}_j^S, \hat{n}_j^S, \hat{\lambda}_j^C] \tag{P}$$

at each s_j. The equations are

$$\hat{q}_j^C = \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{M}_{i,j}\left(1 + \mathrm{MRS}_{i,j}^C\right)\right] \tag{C1}$$

$$\hat{q}_j^S = \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{M}_{i,j}\left(1 - F_{i,j}^S\left(1 - (\pi_B + (1-\pi_B)r_{i,j}^S)\right) + \mathrm{MRS}_{i,j}^S\right)\right] \tag{C2}$$

$$\hat{q}_j^S + \hat{b}_j^S\, q'_S(\hat{b}_j^S) = \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{M}_{i,j}(1 - F_{i,j}^S)\left(1 - \pi_i^R + \frac{\ell_{i,j}}{L_{i,j}^S}\left(\rho_{i,j}^{S,+} + \frac{v_{i,j}^S}{\Pi_{i,j}^S}\right)\right)\right] \tag{C3}$$

$$\hat{p}_j - \hat{q}_j^S\hat{b}_j^S + \phi_K\left(\hat{k}_j^S - 1\right) = \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{M}_{i,j}\Pi_{i,j}^S\,\Omega^S\left(L_{i,j}^S\right)\right] \tag{C4}$$

$$\hat{q}_j^C - \kappa = \hat{\lambda}_j^C + \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{M}_{i,j}(1 - F_{i,j}^C)\right] \tag{C5}$$

$$\hat{p}_j - (\hat{q}_j^C - \kappa)\hat{b}_j^C + \phi_K\left(\hat{k}_j^C - 1\right) = \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{M}_{i,j}\Pi_{i,j}^C\,\Omega^C\left(L_{i,j}^C\right)\right] \tag{C6}$$

$$\hat{\lambda}_j^C\left(\hat{p}_j - (1-\theta)\hat{b}_j^C\right) = 0 \tag{C7}$$

$$1 = \hat{N}_j^H + \hat{N}_j^C + \hat{N}_j^S. \tag{C8}$$

(C1) and (C2) are the household Euler equations for purchases of deposits. (C3) and (C4) are the intertemporal optimality conditions for S-banks, and (C5) and (C6) are those for C-banks. (C7) is the leverage constraint for C-banks. Finally, (C8) is the market clearing condition labor.

Expectations are computed as weighted sums, with the weights being the probabilities of transitioning to exogenous state x_i, conditional on state s_j. Hats ($\hat{\cdot}$) in (C1) – E(C8) indicate variables that are direct functions of the vector of unknowns (P). These are effectively the choice variables for the nonlinear equation solver that finds the solution to the system (C1) – (C8) at each point s_j. All variables in the expectation terms with subscript $_{i,j}$ are direct functions of the forecasting variables (F).

The latter values are *fixed* numbers when the system is solved, as they we pre-computed in step B. For example, the stochastic discount factor $\hat{M}_{i,j}$ depends on both the solution and the forecasting vector, i.e.

$$\hat{M}_{i,j} = \beta \left(\frac{C_{i,j}}{\hat{C}_j} \right)^{-\gamma},$$

since it depends on future and current consumption. To compute the expectation of the right-hand side of equation (C1) at point s_j, we first look up the corresponding column j in the matrix containing the forecasting values that we computed in step B, $\mathscr{F}^m$. This column contains the N^x vectors, one for each possible realization of the exogenous state, of the forecasting values defined in (F). From these vectors, we need consumption $C_{i,j}$. Further, we need current consumption $\hat{C}_j$, which is a policy variable chosen by the nonlinear equation solver. $\text{MRS}^C_{i,j}$ is a function of future consumption $C_{i,j}$, and the future state variables $B^S_{i,j}$ and $B^C_{i,j}$ (since market clearing implies $A^j_t = B^j_t$ for $j = S, C$). Denoting the probability of moving from current exogenous state x_j to state x_i as $\pi_{i,j}$, we compute the expectation of the RHS of (C1)

$$\mathrm{E}_{s'_{i,j}|s_j} \left[\hat{M}_{i,j} \left(1 + \text{MRS}^C_{i,j} \right) \right] = \sum_{x_i \,|\, x_j} \pi_{i,j} \hat{M}_{i,j} \left(1 + \text{MRS}^C_{i,j} \right).$$

The mapping of solution and forecasting vectors (P) and (F) into the other expressions in equations (C1) – (C8) follows the same principles and is based on the equations in model appendix A. In particular, the system (C1) – (C8) implicitly uses the budget constraints of all agents, and the market clearing conditions for capital and debt of both banks.

Note that we could exploit the linearity of the market clearing condition in (C8) to eliminate one more policy variable, $\hat{n}^S$, from the system analytically. However, in our experience the algorithm is more robust when we explicitly include labor demand of all agents as policy variables, and ensure that these variables stay strictly positive (as required with CD production functions) when solving the system. To solve the system in practice, we use a nonlinear equation solver that relies on a variant of Newton's method, using policy functions $\mathcal{C}^m_P$ as initial guess. More on these issues in subsection I.3 below.

The final output of this step is a $N^S \times 12$ matrix $\mathscr{P}^{m+1}$, where each row is the solution vector $\hat{P}_j$ that solves the system (C1) – (C8) at point s_j.

D. **Update forecasting, transition and policy functions.** Given the policy matrix $\mathscr{P}^{m+1}$ from step B, update the policy function directly to get $\mathcal{C}^{m+1}_P$. All forecasting functions with the exception of the value functions are also equivalent to policy functions. The household value function is updated based on the recursive definition

$$\hat{V}_j^H = U(\hat{C}_j, H_{i,j}) + \beta \mathrm{E}_{s'_{i,j}|s_j} V_{i,j}^H \tag{V}$$

using the same notation as defined above under step C. Note that the value function combines current solutions from $\mathscr{P}^{m+1}$ (step C) for consumption with forecasting values from $\mathscr{F}^m$ (step B). Using these updated value functions, we get $\hat{\mathcal{C}}_F^{m+1}$.

Finally, update transition functions for the endogenous state variables using the following laws of motion, for current state s_j and future exogenous state x_i as defined above:

$$\begin{aligned} K_{i,j}^{m+1} &= \hat{I}_j^C + \hat{I}_j^S + (1-\delta_K)\left(1 - \xi^C F_{i,j}^C \rho_{i,j}^{C,-}\right) K_{i,j}^C \\ &\quad + (1-\delta_K)\left(1 - \xi^S F_{i,j}^S \rho_{i,j}^{S,-}\right)(1-\ell_{i,j}^S) K_{i,j}^S + (1-\underline{\delta}_K)\ell_{i,j}^S K_{i,j}^S \end{aligned} \tag{T1}$$

$$(K_{i,j}^S)^{m+1} = \hat{k}_j^S K_{i,j}^S \tag{T2}$$

$$(B_{i,j}^C)^{m+1} = \hat{B}_j^C \tag{T3}$$

$$(B_{i,j}^S)^{m+1} = \hat{B}_j^S. \tag{T4}$$

(T1) is simply the law of motion for aggregate capital, and (T2) is the definition of capital growth k_t^S. (T3) and (T4) follow directly from the direct mapping of policy into state variable for bank debt. Updating according to (T1) – (T4) gives the next set of functions $\hat{\mathcal{C}}_T^{m+1}$.

E. **Check convergence.** Compute distance measures $\Delta_F = ||\mathcal{C}_F^{m+1} - \mathcal{C}_F^m||$ and $\Delta_T = ||\mathcal{C}_T^{m+1} - \mathcal{C}_T F^m||$. If $\Delta_F < \mathrm{Tol}_F$ and $\Delta_T < \mathrm{Tol}_T$, stop and use $\mathcal{C}^{m+1}$ as approximate solution. Otherwise reset policy functions to the next iterate i.e. $\mathscr{P}^m \to \mathscr{P}^{m+1}$ and reset forecasting and transition functions to a convex combination of their previous and updated values i.e. $\mathcal{C}^m \to \mathcal{C}^{m+1} = D \times \mathcal{C}^m + (1-D) \times \hat{\mathcal{C}}^{m+1}$, where D is a dampening parameter set to a value between 0 and 1 to reduce oscillation in function values in successive iterations. Next, go to step B.

Step 3 Using the numerical solution $\mathcal{C}^* = \mathcal{C}^{m+1}$ from step 2, we simulate the economy for $\bar{T} = T_{ini} + T$ period. Since the exogenous shocks follow a discrete-time Markov chain with transition matrix Π_x, we can simulate the chain given any initial state x_0 using $\bar{T} - 1$ uniform random numbers based on standard techniques (we fix the seed of the random number generator to preserve comparability across experiments). Using the simulated path $\{x_t\}_{t=1}^{\bar{T}}$, we can simulate the associated path of the endogenous state variables given initial state $s_0 = [x_0, K_0, K_0^S, B_0^S, B_0^C]$ by evaluating the transition functions

$$[K_{t+1}, K_{t+1}^S, B_{t+1}^C, B_{t+1}^g H0] = \mathcal{C}_T^*(s_t, x_{t+1}),$$

to obtain a complete simulated path of model state variables $\{s_t\}_{t=1}^{\bar{T}}$. To remove any effect of the initial conditions, we discard the first T_{ini} points. We then also evaluate the policy and forecasting functions along the simulated sample path to obtain a complete sample path $\{s_t, P_t, f_t\}_{t=1}^{\bar{T}}$.

To assess the quality and accuracy of the solution, we perform two types of checks. First, we verify that all state variable realizations along the simulated path are within the bounds of the state variable grids defined in step 1. If the simulation exceeds the grid boundaries, we expand the grid bounds in the violated dimensions, and restart the procedure at step 1. Secondly, we compute relative errors for all equations of the system (C1) – (C8) and the transition functions (T1) – (T4) along the simulated path. For equations involving expectations (such as (C1)), this requires evaluating the transition and forecasting function as in step 2B at the current state s_t. For each equation, we divide both sides by a sensibly chosen endogenous quantity to yield "relative" errors; e.g., for (C1) we compute

$$1 = \frac{1}{\hat{q}_j^C} \mathrm{E}_{s'_{i,j}|s_j} \left[\hat{M}_{i,j} \left(1 + \mathrm{MRS}_{i,j}^C \right) \right],$$

using the same notation as in step 2B. These errors are small by construction when calculated at the points of the discretized state grid $\hat{\mathcal{S}}$, since the algorithm under step 2 solved the system exactly at those points. However, the simulated path will likely visit many points that are between grid points, at which the functions $\mathcal{C}^*$ are approximated by interpolation. Therefore, the relative errors indicate the quality of the approximation in the relevant area of the state space. We report average, median, and tail errors for all equations. If errors are too large during simulation, we investigate in which part of the state space these high errors occur. We then add additional points to the state variable grids in those areas and repeat the procedure.

I.c Implementation

Solving the system of equations. We solve system of nonlinear equations at each point in the state space using a standard nonlinear equation solver (MATLAB's fsolve). This nonlinear equation solver uses a variant of Newton's method to find a "zero" of the system. We employ several simple modifications of the system (C1) – (C8) to avoid common pitfalls at this step of the solution procedure. Nonlinear equation solver are notoriously bad at dealing with complementary slackness conditions associated with a constraint. Judd, Kubler, and Schmedders (2002) discuss the reasons for this and also show how Kuhn-Tucker conditions can be rewritten as additive equations for this purpose. Consider the C-bank's Euler Equation for risk-free debt and the Kuhn-Tucker condition for its leverage constraint:

$$\hat{q}_j^C - \kappa = \hat{\lambda}_j^C + \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{M}_{i,j}(1 - F_{i,j}^C)\right]$$
$$0 = \hat{\lambda}_j^C\left(\hat{p}_j - (1-\theta)\hat{b}_j^C\right)$$

Now define an auxiliary variable $h_j \in \mathcal{R}$ and two functions of this variable, such that $\hat{\lambda}_j^{C,+} = \max\{0, h_j\}^3$ and $\hat{\lambda}_j^{C,-} = \max\{0, -h_j\}^3$. Clearly, if $h_j < 0$, then $\hat{\lambda}_j^{I,+} = 0$ and $\hat{\lambda}_j^{C,-} > 0$, and vice versa for $h_j > 0$. Using these definitions, the two equations above can be transformed to:

$$\hat{q}_j^C - \kappa = \hat{\lambda}_j^{C,+} + \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{M}_{i,j}(1 - F_{i,j}^C)\right]$$
$$0 = \hat{p}_j - (1-\theta)\hat{b}_j^C - \hat{\lambda}_j^{C,-}.$$

The solution variable for the nonlinear equation solver corresponding to the multiplier is h_j. The solver can choose positive h_j to make the constraint binding ($\hat{\lambda}_j^{C,-} = 0$), in which case $\hat{\lambda}_j^{C,+}$ takes on the value of the Lagrange multiplier. Or the solver can choose negative h_j to make the constraint non-binding ($\hat{\lambda}_j^{C,+} = 0$), in which case $\hat{\lambda}_j^{C,-}$ can take on any value that makes (K2) hold.

Similarly, certain solution variables are restricted to positive values due to the economic structure of the problem. For example, given the Cobb-Douglas production function, optimal S-bank capital for next period $\hat{K}_j^S$ is always strictly positive. To avoid that the solver tries out negative capital values (and thus output becomes ill-defined), we use $\log(\hat{K}_j^S)$ as solution variable for the solver. This means the solver can make capital arbitrarily small, but not negative.

Grid configuration. We choose to include the relative capital share of S-banks $\tilde{K}_t^S = K_t^S/K_t$ as state variable instead of borrower debt K_t^S such that the total set of endogenous state variables is $[K_t, \tilde{K}_t^S, B_t^C, B_t^S]$. The reason is that the capital share is more stable in the dynamics of the model than the level, since total capital and S-bank capital are strongly correlated. For similar reasons, we choose to include S-bank and C-bank book leverage $b_t^S = B_t^S / K_t^S$ and $b_t^C = B_t^C / K_t^C$ instead of the levels of debt. For the benchmark case, the grid points in each state dimension are as follows

- Y**:** We discretize Y and Z jointly into a 9-state Markov chain (with three possible realizations for each) using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points $\{Y\}_{j=1}^3$ and $\{Z\}_{j=1}^3$ and the 9×9 Markov transition matrix $\Pi_{Y,Z}$ between them to match the volatility and persistence of GDP growth. This yields the possible realizations for Y: $[0.9869, 1.0000, 1.0132]$, and for Z: $[0.9698, 1.0000, 1.0312]$.
- π^R**:** $[0.0, 0.33]$ (see calibration)
- K**:** $[2.92, 3.04, 3.15, 3.26, 3.39, 3.50]$
- $\tilde{K}^S$**:** $[0.26, 0.28, 0.30, 0.32, 0.34, 0.36, 0.38]$
- b^S**:** $[0.10, 0.218, 0.334, 0.451, 0.568, 0.686, 0.803, 0.92]$
- b^C**:** $[0.87, 0.882, 0.894, 0.906, 0.918, 0.93]$

The total state space grid has 24,192 points. The grid boundaries and the placement of points have to be readjusted for each experiment, since the ergodic distribution of the state variables depends on parameters. Finding the right values for the boundaries is a matter of experimentation.

Generating an initial guess and iteration scheme. To find a good initial guess for the policy, forecasting, and transition functions, we solve the deterministic "steady-state" of the model under the assumption that the bank leverage constraint is binding and no runs are occurring. We then initialize all functions to their steady-state values, for all points in the state space. Note that the only role of the steady-state calculation is to generate an initial guess that enables the nonlinear equation solver to find solutions at (almost) all points during the first iteration of the solution algorithm. In our experience, this steady state delivers a good enough initial guess.

In case the solver cannot find solutions for some points during the initial iterations, we revisit such points at the end of each iteration. We try to solve the system at these "failed" points using as initial guess the solution of the closest neighboring point at which the solver was successful. This method works well to speed up convergence and eventually finds solutions at all points.

To determine convergence, we check absolute errors in the value function of households, (V). Out of all functions we approximate during the solution procedure, it exhibits the slowest convergence. We stop the solution algorithm when the maximum absolute difference between two iterations, and for all points in the state space, falls below 1e-3 and the mean distance falls below 1e-4. For appropriately chosen grid boundaries, the algorithm converges within 120 iterations.

We implement the algorithm in MATLAB and run the code on a high-performance computing (HPC) cluster. As mentioned above, the nonlinear system of equations can be solved in parallel at each point. We parallelize across 28 CPU cores of a single HPC node. The total running time for the benchmark calibration is about 2 hours and 40 minutes.

Simulation. To obtain the quantitative results, we simulate the model for 5,000 periods after a "burn-in" phase of 500 periods. The starting point of the simulation is the ergodic mean of the state variables. As described in detail above, we verify that the simulated time path stays within the bounds of the state space for which the policy functions were computed. We fix the seed of the random number generator so that we use the same sequence of exogenous shock realizations for each parameter combination.

To produce impulse response function (IRF) graphs in Figure 1, we simulate 10,000 different paths of 25 periods each. In the initial period, we set the endogenous state variables to several different values that reflect the ergodic distribution of the states. We use a clustering algorithm to represent the ergodic distribution non-parametrically. We fix the initial exogenous shock realization to mean productivity ($Y = Z = 1$) and no run ($\pi^R = 0$). The "impulse" in the second period is either only a bad productivity shock, or both low productivity and a run shock ($\pi^R = 0.3$). For the remaining 23 periods, the simulation evolves according to the stochastic law of motion of the shocks. In the IRF graphs, we plot the median path across the 10,000 paths given the initial condition. The simulation dynamics in Figure 2 are constructed similarly, with the difference that the economy also experiencing unanticipated changes in model parameters.

Evaluating the solution. Our main measure to assess the accuracy of the solution are relative equation errors calculated as described in step 3 of the solution procedure. Table A reports the median error, the 95^{th} percentile of the error distribution, the 99^{th}, and 100^{th} percentiles during the 5,000 period simulation of the model. Median errors are very small for all equations, with even maximum errors only causing small approximation mistakes. Errors are comparably small for all experiments we report.

Table A: Computational Errors

Equation			**Percentile**		
	50th	75th	95th	99th	Max
C1	5.63519E-05	6.63392E-05	7.44099E-05	8.19743E-05	8.75681E-05
C2	5.75018E-05	6.76531E-05	7.58158E-05	8.3402E-05	8.91697E-05
C3	6.08842E-05	7.15472E-05	7.98797E-05	8.75013E-05	9.34417E-05
C4	1.31404E-05	1.69366E-05	2.46315E-05	3.18816E-05	7.70292E-05
C5	3.86174E-05	4.73236E-05	5.48711E-05	5.93791E-05	6.23922E-05
C6	1.27067E-05	1.61208E-05	2.29494E-05	2.9007E-05	7.50274E-05
C7	0.00022121	0.000285663	0.000344085	0.000411546	0.000454729
C8	0.000126335	0.000157528	0.000174314	0.000175913	0.000211747

The table reports median, 75th percentile, 95th percentile, 99th percentile, and maximum absolute value errors, evaluated at state space points from a 5,000 period simulation of the benchmark model. Each row contains errors for the respective equation of the nonlinear system (C1) – (C8) listed in step 2 of the solution procedure.

II Simple Model

II.a Equilibrium Definition

Equilibrium definition. The equilibrium is a set of quantities $\{C_0, C_1, K_S, K_C, L_S, L_C, S_S, S_C, A_C, A_S\}$ and prices $\{p, q_S, q_C, p_S, p_C\}$, such that households maximize (13) subject to constraints (17) and (18), S-banks maximize (9) and (8), C-banks maximize (9) and (6) subject to (7), and the markets for capital $1 = K_S + K_C$, equity shares (sum to 1) and deposits of both bank types, $A_j = B_j$, clear.

By Walras law, consumption at time 0 is[1]

$$C_0 = 0, \tag{1}$$

and consumption at time 1 is

$$C_1 = K_C \left(\mathrm{E}(\rho_C) - F(L_C)\mathrm{E}(\rho_C \,|\rho_C < L_C) \right) + K_S \left(\mathrm{E}(\rho_S) - F(L_S)\mathrm{E}(\rho_S \,|\rho_S < L_S) \right). \tag{2}$$

The resource constraint for period-1 consumption (2) clarifies the fundamental welfare trade-off of the model. If banks did not issue any deposits, then $L_C = L_S = 0$, no bank would default, and household consumption of the numeraire good would be maximized at the full payoff of capital, $\mathrm{E}(\rho_j)$. However, in that case banks would produce no liquidity

[1]The funds households spend on their portfolio of bank securities, $q_C A_C + p_C S_C + q_S A_S + p_S S_S$, are equal to the market value of the capital they sell to banks in equilibrium, p.

services from which households also derive utility. To produce liquidity services, banks need to issue deposits and take on leverage, which causes a fraction $F_j(L_j)$ of them to default. In the process, some payoffs of the numeraire good are destroyed.

II.b Preliminary Definitions

To unify notation in all following proofs, we first define the ratio of S-Bank to C-Bank deposits

$$R_S = \frac{A_S}{A_C}.$$

We compute the partial derivatives of the liquidity utility function in equation (19)

$$\mathcal{H}_j(A_S, A_C) = \frac{\partial H(A_S, A_C)}{\partial A_j} = \left(\alpha A_S^{\epsilon} + (1-\alpha) A_C^{\epsilon}\right)^{-\gamma/\epsilon} \tilde{\mathcal{H}}_j(R_S), \tag{3}$$

for $j = S, C$, and where $\tilde{\mathcal{H}}_j$ denote the partial derivatives if $\gamma = 0$:

$$\tilde{\mathcal{H}}_S(R_S) = \left.\frac{\partial H(A_S, A_C)}{\partial A_S}\right|_{\gamma=0} = \alpha \left(\alpha + (1-\alpha)\left(\frac{1}{R_S}\right)^{\epsilon}\right)^{\frac{1-\epsilon}{\epsilon}} \tag{4}$$

$$\tilde{\mathcal{H}}_C(R_S) = \left.\frac{\partial H(A_S, A_C)}{\partial A_C}\right|_{\gamma=0} = (1-\alpha)\left(\alpha R_S^{\epsilon} + (1-\alpha)\right)^{\frac{1-\epsilon}{\epsilon}} \tag{5}$$

The derivatives conditional on $\gamma = 0$ only depend on the ratio R_S, whereas the full partials also depend on the levels of C-bank and S-bank deposits.

II.c C-bank and S-bank problem: size and leverage choice

For C-banks, the leverage problem is

$$v_C = \max_{L_C \in [0,1]} \; q_C L_C - p + \beta \left(1 - F_C(L_C)\right)\left(\rho_C^{+} - L_C\right) \tag{6}$$

subject to

$$L_C \leq (1-\theta)\, \mathrm{E}(\rho_C), \tag{7}$$

and for S-banks it is

$$v_S = \max_{L_S \in [0,1]} \; q_S(L_S) L_S - p + \beta \left(1 - F_S(L_S)\right)\left(\rho_S^{+} - L_S\right). \tag{8}$$

The capital purchase decision for each bank is then given by

$$\max_{K_j \geq 0} K_j v_j. \tag{9}$$

Each individual S-bank recognizes that the price of its debt is a function of its leverage according to households' valuation in (23). However, S-banks are price takers and do not internalize the effect of their leverage choice on the *aggregate* marginal benefit of S-bank liquidity $\psi\mathcal{H}_S(A_S, A_C)$. The following proposition characterizes S-banks' optimizing behavior, denoting by f_S the density of distribution F_S.

Proposition 1. *1. S-bank marginal defaults are equal to the marginal benefit of S-bank liquidity:*

$$L_S f_S(L_S) = \psi\mathcal{H}_S(A_S, A_C). \tag{10}$$

2. S-banks' demand for capital implies the following restriction on the capital price:

$$p = \beta\left((1 - F_S(L_S))\rho_S^+ + \psi L_S \mathcal{H}_S(A_S, A_C)\right).$$

Proof. To obtain the S-bank FOC for leverage, we differentiate the S-bank objective in (8) to get

$$q_S + q_S'(L_S)L_S = \beta(1 - F_S(L_S)).$$

Differentiating the HH FOC (23) with respect to L_S, and under the restriction that individual S-banks do not internalize their effect on aggregate S-bank liquidity A_S, gives

$$q_S'(L_S) = -\beta f_S(L_S).$$

Combining the two yields

$$q_S = \beta(1 - F_S(L_S) + f_S(L_S)L_S). \tag{11}$$

Substituting this result back into the HH FOC (23) results in equation (10) for part 1. For part 2., we first note that a positive amount of S-bank capital $K_S > 0$ in equilibrium requires that the expected profit per unit is zero, $v_S = 0$, which when combined with equation (8) gives

$$p = q_S L_S + \beta(1 - F(L_S))(\rho_S^+ - L_S).$$

Substituting for q_S from (11) and (10), and simplifying gives the result. □

Proposition 1 states that S-banks optimally choose leverage such that the marginal benefit of S-bank liquidity to households, on the RHS of (10), is equal to the marginal loss due to defaulting S-banks (LHS).

Further, because of constant returns to scale and competitive markets, S-banks must have zero expected value in equilibrium. This restriction leads to equation (25), which states that S-bank demand for capital is perfectly elastic at a price p.

Turning to C-banks, the following proposition characterizes their optimal choices.

Proposition 2. *If there is a positive marginal benefit of C-bank liquidity, $\psi\mathcal{H}_C(A_S, A_C) > 0$, the C-bank leverage constraint is always binding, implying $L_C = E(\rho_C)(1-\theta)$, and C-banks' capital demand requires*

$$p = \beta\left((1 - F_C(L_C))\rho_C^+ + \psi L_C \mathcal{H}_C(A_S, A_C) + F_C(L_C)L_C\right).$$

Proof. Differentiating the C-bank objective in (6) with respect to L_C gives

$$q_C = \mu_C + \beta(1 - F_C(L_C)), \tag{12}$$

where μ_C is the Lagrange multiplier on the leverage constraint. Combining equations (22) and (12) yields

$$\mu_C = \beta(\psi\mathcal{H}_C + F_C(L_C)) > 0. \tag{13}$$

Under the assumption that $\psi\mathcal{H}_C > 0$, this implies that the multiplier is positive and the constraint is binding, with leverage given by

$$L_C = \mathrm{E}(\rho_C)(1-\theta).$$

Like for S-Banks, a positive amount of C-Bank capital $K_C > 0$ requires zero expected profit per unit of capital $v_C = 0$, which by equation (6) implies

$$p = q_C L_C + \beta(1 - F(L_C))(\rho_C^+ - L_C).$$

Substituting for q_C from the household FOC for C-Bank deposits (22) gives the result in (24). □

Since C-banks can issue insured debt that also generates utility for households, there is no interior optimum to their capital structure choice. Analogous to S-banks, the scale invariance of the C-bank problem requires that C-banks make zero profits. Combining this condition with the price of C-bank debt required by HH optimization in (22) gives the equation in (24).

II.d Proofs for Main Text

Proof of proposition 1. The proposition assumes identical distributions for bank-idiosyncratic shocks, i.e. $F_S = F_C = F$. Then the first-order conditions for S- and C-bank leverage are given by

$$f(L_j)L_j = \psi \mathcal{H}_j(L_S K_S, L_C(1-K_S)) \tag{14}$$

for $j = S, C$ respectively. The first-order condition for the S-bank capital share is

$$(1-F(L_S))(\rho_S^+ - L_S) - L_S \psi \mathcal{H}_S(L_S K_S, L_C(1-K_S)) = (1-F(L_C))(\rho_C^+ - L_C) - L_C \psi \mathcal{H}_C(L_S K_S, L_C(1-K_S)) \tag{15}$$

We conjecture and verify that the optimal allocation features equal leverage

$$L_S = L_C.$$

Under this assumption, (14) imply $\tilde{\mathcal{H}}_S(R_S) = \tilde{\mathcal{H}}_C(R_S)$, using the definition from (4) and (5), or

$$\alpha \left[\alpha + (1-\alpha) \frac{1}{R_S^{\epsilon}} \right]^{\frac{1-\epsilon}{\epsilon}} = (1-\alpha) \left[\alpha R_S^{\epsilon} + 1 - \alpha \right]^{\frac{1-\epsilon}{\epsilon}}.$$

It is easy to verify that the solution to this equation is

$$R_S = \left(\frac{\alpha}{1-\alpha} \right)^{\frac{1}{1-\epsilon}},$$

implying $\tilde{\mathcal{H}}_S(R_S) = \tilde{\mathcal{H}}_C(R_S) = 1$. Given this solution, we indeed get that $L_S = L_C$ as conjectured from (14).

Since

$$R_S = \frac{L_S K_S}{L_C K_C} = \frac{K_S}{K_C},$$

we obtain the solution for the capital shares in the proposition.

Plugging this solution back into either condition (14) gives an implicit equation for optimal leverage

$$L^*f(L^*) = \left[\psi(1-\alpha)\left(\alpha\left(\frac{\alpha}{1-\alpha}\right)^{\frac{\epsilon}{1-\epsilon}}+1-\alpha\right)^{\frac{1-\epsilon-\gamma}{\epsilon}}\left(\frac{(1-\alpha)^{1/(1-\epsilon)}}{\alpha^{1/(1-\epsilon)}+(1-\alpha)^{1/(1-\epsilon)}}\right)^{-\gamma}\right]^{\frac{1}{1+\gamma}}. \tag{16}$$

Proof of proposition 2.

Proof. This proposition assumes that bank-idiosyncratic shocks are distributed Uniform$[0,1]$. Given this assumption, we can write the S-bank leverage condition as

$$L_S = \psi\mathcal{H}_S(A_S, A_C), \tag{17}$$

and the definition of the liquidity wedge m as

$$L_C = \psi(1+m)\mathcal{H}_C(A_S, A_C). \tag{18}$$

Further, the capital market condition (26) simplifies to

$$L_S^2 = L_C^2 + 2L_C\psi\mathcal{H}_C\left(A_S, A_C\right), \tag{19}$$

which combined with (17) and (18) gives

$$L_S = \sqrt{\frac{m+3}{m+1}}L_C. \tag{20}$$

Part (i) follows directly from this relation: for any value of $m \in (-1, \infty)$ we have $L_S > L_C$. In particular, at the "planner solution" of $m = 0$, we have $L_S = \sqrt{3}L_C$.

For part (ii), combining (20) with (17) and (18), we get the following equation in m and the deposit ratio R_S

$$\tilde{\mathcal{H}}_S(R_S) = \sqrt{(m+1)(m+3)}\tilde{\mathcal{H}}_C(R_S),$$

using the definitions in (4) and (5). Defining the wedge factor $\mathcal{M} = \sqrt{(m+1)(m+3)}$ this can be written as

$$\alpha\left[\alpha + (1-\alpha)\frac{1}{R_S^{\epsilon}}\right]^{\frac{1-\epsilon}{\epsilon}} = \mathcal{M}(1-\alpha)\left[\alpha R_S^{\epsilon} + 1 - \alpha\right]^{\frac{1-\epsilon}{\epsilon}},$$

which can be rearranged to

$$\alpha(1-\alpha)^{\frac{\epsilon}{1-\epsilon}}\mathcal{M}^{\frac{\epsilon}{1-\epsilon}}R_S^{2\epsilon}+\left((1-\alpha)^{\frac{1}{1-\epsilon}}\mathcal{M}^{\frac{\epsilon}{1-\epsilon}}-\alpha^{\frac{1}{1-\epsilon}}\right)R_S^{\epsilon}-(1-\alpha)\alpha^{\frac{\epsilon}{1-\epsilon}}=0.$$

This is an exponential polynomial of the form

$$\mathcal{A}\exp(2\epsilon x)+\mathcal{B}\exp(\epsilon x)+\mathcal{C}=0,$$

with $x=\log(R_S)$ and

$$\begin{aligned}\mathcal{A}&=\alpha(1-\alpha)^{\frac{\epsilon}{1-\epsilon}}\mathcal{M}^{\frac{\epsilon}{1-\epsilon}}\\ \mathcal{B}&=\left((1-\alpha)^{\frac{1}{1-\epsilon}}\mathcal{M}^{\frac{\epsilon}{1-\epsilon}}-\alpha^{\frac{1}{1-\epsilon}}\right)\\ \mathcal{C}&=-(1-\alpha)\alpha^{\frac{\epsilon}{1-\epsilon}}.\end{aligned}$$

Given $\alpha\in[0,1]$, the unique real root is

$$x=\frac{1}{\epsilon}\log\left[\frac{(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\epsilon}}}{\alpha\mathcal{M}^{\frac{\epsilon}{1-\epsilon}}}\right],$$

which simplifies to

$$R_S=\frac{A_S}{A_C}=\left(\frac{1}{\mathcal{M}}\right)^{\frac{1}{1-\epsilon}}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\epsilon}},$$

which is equation (28) in the main text. To get the result for the capital ratio K_S/K_C in equation (28), note that

$$R_S=\frac{L_SK_S}{L_CK_C}=\sqrt{\frac{m+3}{m+1}}\frac{K_S}{K_C},$$

using (20), which implies

$$\frac{K_S}{K_C}=\frac{1+m}{\mathcal{M}}R_S.$$

To prove part (iii), suppose the regulator in the competitive equilibrium can choose θ such that $m=0$, which yields the planner solution for C-bank leverage $L_C=\psi\mathcal{H}_C(A_S,A_C)$ (the proof to proposition 3 below establishes that there is unique mapping between θ and m). Choosing $m=0$ implies $\mathcal{M}=\sqrt{3}$, which yields a smaller S-bank ratio R_S by factor $1/\sqrt{3}^{\frac{1}{1-\epsilon}}$ compared to the planner solution in proposition 1. This proves that there is no competitive equilibrium (for any value of θ and the other parameters) that simultaneously satisfies optimal leverage and S-bank share in the planner solution of proposition 1. □

The following lemma establishes that there is a unique mapping between the capital requirement θ and the liquidity wedge m in the competitive equilibrium.

Lemma 3. *The liquidity wedge m is strictly decreasing in the capital requirement θ everywhere, i.e.,* $\frac{dm}{d\theta} < 0$.

Proof. Since $L_C = \frac{1}{2}(1-\theta)$ and thus $\frac{d\theta}{dL_C} < 0$, we prove the result by showing that $\frac{dL_C}{dm} > 0$. We start by substituting the definition of $\mathcal{H}_C$ from (3) into the optimality condition for C-bank leverage (27) to obtain

$$L_C = \psi(1+m)(1-\alpha)\left(\alpha R_S^{\epsilon} + (1-\alpha)\right)^{\frac{1-\epsilon-\gamma}{\epsilon}} A_C^{-\gamma}.$$

Since $A_C = L_C K_C$ and by (28)

$$K_C = \frac{\mathcal{M}}{\mathcal{M} + (1+m)R_S},$$

we can solve for $L_C^{1+\gamma}$ as a function of m and other parameters

$$L(m) \equiv L_C^{1+\gamma} = \psi(1-\alpha)(1+m)\mathcal{M}^{-\gamma}\left(\alpha R_S^{\epsilon} + (1-\alpha)\right)^{\frac{1-\epsilon-\gamma}{\epsilon}}\left(\mathcal{M} + (1+m)R_S\right)^{\gamma},$$

where $\mathcal{M} = \sqrt{(1+m)(3+m)}$ and R_S are also functions of m. To establish that $\frac{dL_C}{dm} > 0$, it suffices to show that $L'(m) > 0$. Differentiating and collecting terms, we get

$$L'(m) = L_m^1 + L_m^2,$$

where

$$L_m^1 = \psi\mathcal{M}^{-\gamma}\left(\alpha R_S^{\epsilon} + (1-\alpha)\right)^{\frac{1-2\epsilon-\gamma}{\epsilon}}\left(1-\alpha+\frac{\alpha}{3+m}R_S^{\epsilon}\right),$$

and

$$L_m^2 = 2\gamma\frac{\alpha R_S^{\epsilon}\left(6+m(5+m)+(1-\epsilon)R_S\mathcal{M}\right) - (1-\alpha)(1+m+\epsilon)R_S\mathcal{M}}{(1+m)(3+m)(3+m+R_S\mathcal{M})}.$$

Clearly, $L_m^1 > 0$ for $\alpha \in [0,1]$. Thus $L_m^2 > 0$ is a sufficient condition for $L'(m) > 0$. $L_m^2 > 0$ if its numerator is positive:

$$\alpha R_S^{\epsilon}\left(6+m(5+m)+(1-\epsilon)R_S\mathcal{M}\right) - (1-\alpha)(1+m+\epsilon)R_S\mathcal{M} > 0.$$

To verify this condition, first note that

$$R_S^{\epsilon} = \frac{1-\alpha}{\alpha}R_S\mathcal{M},$$

such that the condition simplifies to

$$6 + m(5 + m) + (1 - \epsilon)R_S\mathcal{M} - (1 + m + \epsilon) > 0.$$

Since $R_S\mathcal{M} > 0$ and $\epsilon \leq 1$, we can further reduce the condition to

$$m^2 + 4m + 5 > \epsilon.$$

Since m is bounded from below by -1, the left-hand side is bounded from below by 2. Since $\epsilon \leq 1$, the right-hand side is bounded from above by 1. Thus the condition is globally satisfied, proving that

$$\frac{dL_C}{dm} > 0 \Leftrightarrow \frac{dm}{d\theta} < 0.$$

□

Proof of proposition 3.

Proof. Part (1.i) follows directly from the binding bank leverage constraint $L_C = \frac{1}{2}(1 - \theta)$.

For part (1.ii), recall that

$$R_S = \frac{A_S}{A_C} = \left(\frac{1}{\mathcal{M}}\right)^{\frac{1}{1-\epsilon}} \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\epsilon}}.$$

We differentiate this expression with respect to m

$$\frac{dR_S}{dm} = -\frac{(2+m)R_S}{(1-\epsilon)\mathcal{M}},$$

implying $dR_S/dm < 0$. Since by lemma 3 above, $dm/d\theta < 0$, we have $dR_S/d\theta > 0$.

For the capital ratio, recall

$$\frac{K_S}{K_C} = \frac{L_S A_S}{L_C A_C} = \sqrt{\frac{1+m}{3+m}} R_S,$$

by equation (28). We again differentiate with respect to m

$$\frac{dK_S/K_C}{dm} = -\frac{(1+m+\epsilon)R_S}{(1-\epsilon)(3+m)\mathcal{M}},$$

which implies that $d(K_S/K_C)/dm < 0$. Again combining this with $dm/d\theta < 0$ by lemma

3, we get $d(K_S/K_C)/d\theta > 0$.

For part (1.iii), first recall that from the S-Bank first-order condition for leverage we have

$$L_S = \psi \mathcal{H}_j(A_S, A_C).$$

Based on the definition of $\mathcal{H}_j(A_S, A_C)$ in (3) this can be written as

$$L_S = \psi\alpha \left(\alpha + (1-\alpha)R_S^{\epsilon}\right)^{\frac{1-\epsilon-\gamma}{\epsilon}} A_S^{-\gamma}.$$

Using $A_S = K_S L_S$ and

$$K_S = \frac{(1+m)R_S}{\mathcal{M} + (1+m)R_S},$$

this can be expressed as function of m and parameters

$$\hat{L}_S = L_S^{1+\gamma} = \psi\alpha \left(\alpha + (1-\alpha)R_S^{\epsilon}\right)^{\frac{1-\epsilon-\gamma}{\epsilon}} \left((1+m)R_S\right)^{-\gamma}(\mathcal{M} + (1+m)R_S)^{\gamma}.$$

We differentiate with respect to m

$$\frac{d\hat{L}_S}{dm} = \frac{L_m^1 L_m^2}{L_m^3},$$

with

$$L_m^1 = (1+m)^{1-\gamma}\mathcal{M}^{\frac{\epsilon}{1-\epsilon}-2}R_S^{\epsilon-\gamma}\left(\mathcal{M} + (1+m)R_S\right)^{\gamma}\left(\alpha + (1-\alpha)R_S^{-\epsilon}\right)^{\frac{1-\gamma}{\epsilon}},$$
$$L_m^2 = \gamma(3+m)(1+m+\epsilon)R_S^{\epsilon} + (1-\alpha)\left((1-\epsilon)(2+m-\gamma)(3+m) + (1-\epsilon-\gamma)(2+m)R_S\mathcal{M}\right), \text{and}$$
$$L_m^3 = (1-\epsilon)\left(1-\alpha+\alpha R_S^{\epsilon}\right)^2 \mathcal{M}^{\frac{1}{1-\epsilon}}\left(1 + (1+m)R_S\right).$$

Since $L_m^1 > 0$ and $L_m^3 > 0$ for all parameter values, the sign of the derivative depends on the sign of L_m^2. This expression can be positive or negative, depending on parameters. In particular, it can be negative if γ is large. The following result proves Corollary 1 in the main text: if $\gamma_H = 0$, we get

$$L_m^2|_{\gamma_H=0} = (1-\alpha)(1-\epsilon)(2+m)(3+m+R_S\mathcal{M}) > 0.$$

Thus for $\gamma_H = 0$, we have that $\frac{dL_S}{dm} > 0$ and $\frac{dL_S}{d\theta} < 0$.

For part 2., we differentiate the household objective given by (20) in the *decentralized* *equilibrium* with respect to θ. After collecting terms, the derivate is

$$\frac{dU(\theta)}{d\theta} = \frac{dK_C}{d\theta}\left(\psi\mathcal{H}_C(A_S, A_C)L_C + (1 - F_C(L_C))\rho_C^+\right) + \frac{dK_S}{d\theta}\left(\psi\mathcal{H}_S(A_S, A_C)L_S + (1 - F_C(L_S))\rho_S^+\right)$$
$$+ \frac{dL_C}{d\theta}\left(\psi\mathcal{H}_C(A_S, A_C)K_C - K_C L_C f_C(L_C)\right) + \frac{dL_S}{d\theta}\left(\psi\mathcal{H}_S(A_S, A_C)K_S - K_S L_S f_S(L_S)\right).$$

Since in equilibrium $L_S f_S(L_S) = \psi\mathcal{H}_S(A_S, A_C)$ and $L_C f_C(L_C) = (1+m)\psi\mathcal{H}_C(A_S, A_C)$, this expression becomes

$$\frac{dU(\theta)}{d\theta} = \frac{dK_C}{d\theta}\left(\psi\mathcal{H}_C(A_S, A_C)L_C + (1 - F_C(L_C))\rho_C^+\right) + \frac{dK_S}{d\theta}\left(\psi\mathcal{H}_S(A_S, A_C)L_S + (1 - F_C(L_S))\rho_S^+\right)$$
$$- m\psi\mathcal{H}_C(A_S, A_C)K_C\frac{dL_C}{d\theta}.$$

Further noting that $dK_C/d\theta = -dK_S/d\theta$ (since $K_C = 1 - K_S$), applying the capital market condition (26), and noting that $dL_C/d\theta = -\mathrm{E}(\rho_C)$ gives expression (29) in the main text. Since $dK_S/d\theta > 0$ by part (1.ii) and $F_C(L_C)L_C > 0$, this expression is positive for any $m \geq 0$.

□

III Calibration Appendix

III.a Bank idiosyncratic shocks

In the model, we parameterize the idiosyncratic ρ shocks as gamma distributions. Let the gamma cumulative distribution function be given by $\Gamma(\rho;\, \chi_0, \chi_1)$ with parameters (χ_0, χ_1). These parameters map into means μ_ρ^j and variances $\sigma_{\rho^j}^2$ as follows:

$$\chi_1 = \sigma_\rho^2/\mu_\rho,$$
$$\chi_0 = \mu_\rho/\chi_1.$$

A standard result in statistics states that the conditional expectations are

$$\mathrm{E}(\rho \,|\, \rho < x) = \mu_\rho \frac{\Gamma(x;\, \chi_0 + 1, \chi_1)}{\Gamma(x;\, \chi_0, \chi_1)},$$
$$\mathrm{E}(\rho \,|\, \rho > x) = \mu_\rho \frac{1 - \Gamma(x;\, \chi_0 + 1, \chi_1)}{1 - \Gamma(x;\, \chi_0, \chi_1)},$$

which we use to compute the conditional expectations $\rho^{j,-}$ and $\rho^{j,+}$ used in bank payoffs to shareholders and recovery values for creditors.

III.b Detailed calibration description

The main text focussed only on the five parameters $(\beta, \alpha, \psi, \gamma_H, \epsilon)$ that govern households' liquidity preferences. This appendix subsection discusses the calibration strategy for all remaining parameters.

The parameters of our model belong to one of two groups. We can set parameters of the first group (listed in Panel A of Table 1) in isolation of any other parameters, i.e., there is a one-to-one mapping between target moment in the data and corresponding model parameter. The second group involves parameters listed in Panel B of Table 1 that we choose jointly to match moments of the ergodic distribution in our model to the corresponding moments in the data. We start with a guess for the parameter values, solve the model with these values, then calculate the moments from the ergodic distribution, and compare these moments to the data. We iterate until the targeted moments in Panel B of Table 1 closely match the data.

Using our definition of bank-dependent sector output, we can calculate the volatility and autocorrelation (ρ^Y) of the bank-independent sector output growth rate and back out σ^Y. Given σ^Y, we set σ^Z to match the volatility of bank-dependent firms' output growth. We calibrate ν^Z, the scale of the bank dependent sector productivity shock, to target the share of bank-dependent real GDP per capita in total GDP.[2]

Our model has two types of adjustment costs: investment and capital growth adjustment costs. They are governed by the parameters ϕ_I and ϕ_K, respectively. The value of ϕ_I determines the marginal cost of investment and therefore the investment volatility of the bank dependent sector in the model. We use the volatility of 2.65% of the logged and HP-filtered investment-asset time series as our target. We introduce capital growth adjustment costs in the model to reflect frictions in the capital flow between shadow- and commercial banks. Hence, the asset growth volatility of either bank type should be informative about ϕ_K. Because it is straightforward to obtain, we choose to the asset growth volatility of commercial banks as a target. We deflate this series, express it in per capita terms, and calculate a quarterly growth rate of 0.5%. Based on NIPA data, we set δ_K to

[2] In the model, we calculate the bank-dependent GDP share as

$$\left(Y_t^C + Y_t^S\right) / \left(Y_t^C + Y_t^S + Y_t^H + Y_t\right).$$

match the depreciation rate of the capital stock to 2.5% per quarter. We set η to 0.667, the labor share in production.

We set the regulatory capital ratio θ in the baseline model to commercial banks' aggregate Tier-1 equity ratio 10%. Although the regulatory minimum ratio is lower in the data, banks tend to keep a small capital buffer, presumably to withstand small shocks without immediately risking supervisory action. To calibrate the deposit insurance fee κ_C, we use the 2016 FDIC report that states that banks paid \$10 billion in FDIC insurance fees on an insurance fund balance of \$83.162 billion. This represents 1.18% of insured deposits, implying a κ_C of 14.2 basis points per dollar of insured deposits.[3]

Banks' default behavior is predominately governed by five parameters (δ_j, ξ_j, with $j \in \{C, S\}$, and π_B). The non-pecuniary default penalties δ_j determine default thresholds of both types of banks. Typically, the default threshold is assumed to be zero with the reasoning that default occurs whenever equity holders are wiped out. However, distressed firms' franchise value is often difficult to measure. Rather than assuming a zero threshold, we use default rates in the data to inform our choice of δ_j. To calibrate the default rates, we use commercial banks' average quarterly loan net-charge off ratio of 0.23% and the quarterly default rate on non-bank financial bond defaults of 0.28% as targets. The bankruptcy costs parameters ξ_j with $j \in \{C, S\}$ determine how much of banks' asset value can be recovered to pay out their creditors in case of default. For commercial banks, we target the recovery value on senior secured debt and loans of 71.9% (from Moody's) net of an additional loss of 33.18% due to the FDIC's resolution costs. This means that our target for the total recovery value on commercial bank debt amounts to 48.1%. For shadow banks' recovery value, we target the average recovery value of 38.1% of senior unsecured debt and subordinated debt.

Using our data definition of banks' valuation shocks ρ_j, t, we parameterize each bank type's Gamma distribution with the standard deviation that we set to the time-series average of the cross-sectional standard deviation of each bank type's equity payout per share. This results in 12.1% for commercial banks and 25.4% for shadow banks. The leverage of shadow banks is informative about the shadow bailout probability parameter π_B. A higher value of π_B means that a large fraction of S-bank debt is insured. For this reason, creditors do not fully price the default risk of S-banks, lowering S-banks' incentives to internalize default costs. S-banks can then increase their equity valuation by increasing leverage. Hence, we use S-bank leverage of 87% as a target for π_B.[4]

[3] `https://www.fdic.gov/about/strategic/report/2016annualreport/ar16section3.pdf`

[4] Note that our shadow bank definition includes GSEs that tend to be very highly levered. Finance companies, also included in our definition of shadow banks, have typically lower leverage ratios.

The behavior of runs is governed by several parameters in the model. We match the bank asset payoffs during the run state to 26% based on Campbell, Giglio, and Pathak (2011), and the fraction of households that run on banks during a run state to 0.333 consistent with Covitz, Liang, and Suarez (2013). We set the run state probabilities such that (i) the unconditional run probability matches the occurrence of banking panics over our sample period and (ii) the average length of the run state equals just over a quarter. The remaining parameter to be determined is the depreciation rate of the capital stock during a run state $\underline{\delta}_K$. This parameter is important for the discount rate on assets during the run state. To determine this parameter, we pick the average haircut of 15.1% documented by Gorton and Metrick (2009) as a target.[5]

III.c Derivation of liquidity spread regression

To motivate our regression design for calibrating ϵ and γ_H, we derive an equation for the model spread between the rate on S-bank and C-bank debt. The starting point are the household first-order conditions for holdings of the two types of debt, (47) and (48), under the simplifying assumptions that $\pi_B = 0$ and $\pi^R_{t+1} = 0$ (these assumptions do not affect the fundamental conclusions from the derivation). Since we are looking for a simple empirical relationship, we further suppress the expectations operators. Under these assumptions, the equations are

$$q^C_t = M_{t,t+1}\left(1 + \mathrm{MRS}^C_{t+1}\right)$$
$$q^S_t = M_{t,t+1}\left(1 - F^S_{\rho,t+1} + F^S_{\rho,t+1} r^S_{t+1} + \mathrm{MRS}_{S,t+1}\right),$$

where

$$\mathrm{MRS}_{S,t} = \alpha\psi C^\gamma_t H^{-\gamma_H}_t \left(\frac{H_t}{A^S_t}\right)^{1-\epsilon}, \tag{21}$$

$$\mathrm{MRS}_{C,t} = (1-\alpha)\psi C^\gamma_t H^{-\gamma_H}_t \left(\frac{H_t}{A^C_t}\right)^{1-\epsilon}, \tag{22}$$

and

$$M_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

We perform a first-order log-linear expansion of both conditions around the deterministic steady state of the model. Variables without time subscript and a bar ($\bar{x}$) denote steady

[5] See the haircut for various asset classes during the crisis in Figure 2 in Gorton and Metrick (2009)

state values, and hatted ($\hat{x}$) variables denote log-deviations from steady state. The usual log-linearization techniques give

$$\hat{q}_t^C = -\gamma \hat{C}_{t+1} + \frac{\beta M\bar{R}S^C}{\bar{q}^C} \mathrm{M\hat{R}S}_{t+1}^C,$$

and

$$\hat{q_t^S} = \frac{\beta}{\bar{q}^S}\left[\left(1-\bar{F}^S+\bar{F}^S\hat{r}^S+M\bar{R}S_{t+1}^S\right)\hat{M}_{t+1}-\bar{F}^S\left(1-\hat{r}^S\right)\hat{F}_{\rho,t+1}^S+\bar{F}^S\hat{r}^S\hat{r}_{t+1}^S\right]....$$
$$+\frac{\beta}{\bar{q}^S}M\bar{R}S^S\left(\gamma\hat{C}_{t+1}+(1-\epsilon-\gamma_H)(1-\alpha)\frac{\bar{A}_C^\epsilon}{\bar{H}^\epsilon}\hat{A}_{C,t+1}+\left((1-\epsilon-\gamma_H)\alpha\frac{\bar{A}_S^\epsilon}{\bar{H}^\epsilon}+(\epsilon-1)\right)\hat{A}_{t+1}^S\right).$$

Further expanding

$$\mathrm{M\hat{R}S}_t^j = \gamma\hat{C}_t + (1-\epsilon-\gamma_H)\hat{H}_t - (1-\epsilon)\hat{A}_t^j$$

and

$$\hat{H}_t = \alpha\frac{(A^S)^\epsilon}{H^\epsilon}\hat{A}_t^S + (1-\alpha)\frac{(A^C)^\epsilon}{H^\epsilon}\hat{A}_t^C,$$

we can compute the spread $\hat{q}_t^C - \hat{q}_t^S$ and collect terms to get

$$\hat{q_{C,t}} - \hat{q_{s,t}} = \hat{M}_{t+1}\left(\frac{\beta}{\bar{q}^C}\left(1+M\bar{R}S_{t+1}^C\right)-\frac{\beta}{\bar{q}^S}\left(1-\bar{F}^S+\bar{F}^S\hat{r}^S+M\bar{R}S_{t+1}^S\right)\right)...$$
$$+\beta\gamma\left(\frac{M\bar{R}S^C}{\bar{q}^C}-\frac{M\bar{R}S^S}{\bar{q}^S}\right)\hat{C}_{t+1}+\frac{\beta}{\bar{q}^S}\bar{F}^S\left(1-\hat{r}^S\right)\hat{F}_{\rho,t+1}^S-\frac{\beta}{\bar{q}^S}\bar{F}^S\hat{r}^S\hat{r}_{t+1}^S...$$
$$+\beta\left(\frac{M\bar{R}S^C}{\bar{q}^C}(\epsilon-1)+\left(\frac{M\bar{R}S^C}{\bar{q}^C}-\frac{M\bar{R}S^S}{\bar{q}^S}\right)(1-\epsilon-\gamma_H)(1-\alpha)\frac{A_C^\epsilon}{H^\epsilon}\right)\hat{A}_{C,t+1}...$$
$$+\beta\left(\left(\frac{M\bar{R}S^C}{\bar{q}^C}-\frac{M\bar{R}S^S}{\bar{q}^S}\right)(1-\epsilon-\gamma_H)\alpha\frac{A_S^\epsilon}{H^\epsilon}-\frac{M\bar{R}S^S}{\bar{q}^S}(\epsilon-1)\right)\hat{A}_{S,t+1}. \quad (23)$$

The coefficients in front of the liquidity quantities $\hat{A}_{t+1}^S$ and $\hat{A}_{t+1}^C$ in equation (23) reveal the role of γ_H and ϵ for the effect of debt quantities on the spread. Clearly, if $\epsilon = 1$ (perfect substitutes) and $\gamma_H = 0$ (constant returns in total liquidity), the liquidity quantity terms drop out and thus do not affect the spread. If $\epsilon = 1$ and $\gamma_H > 0$, the equation becomes

$$
\begin{aligned}
\hat{q_{C,t}} - \hat{q_{s,t}} = \hat{M}_{t+1} \left(\frac{\beta}{\bar{q}^C} \left(1 + M\bar{R}S^C_{t+1}\right) - \frac{\beta}{\bar{q}^S} \left(1 - \bar{F}^S + \bar{F}^S \hat{r}^S + M\bar{R}S^S_{t+1}\right)\right) ... \\
+ \beta\gamma \left(\frac{M\bar{R}S^C}{\bar{q}^C} - \frac{M\bar{R}S^S}{\bar{q}^S} \right) \hat{C}_{t+1} + \frac{\beta}{\bar{q}^S} \bar{F}^S \left(1 - \hat{r}^S\right) \hat{F}^S_{\rho,t+1} - \frac{\beta}{\bar{q}^S} \bar{F}^S \hat{r}^S \hat{r}^S_{t+1} ... \\
- \beta\gamma_H \left(\frac{M\bar{R}S^C}{\bar{q}^C} - \frac{M\bar{R}S^S}{\bar{q}^S} \right) \hat{H}_{t+1}, \qquad (24)
\end{aligned}
$$

i.e., in this case it is only the total quantity of liquidity services $\hat{H}_t$ that matters but not the type of liquidity services.

For simplicity further assume that $\xi^S = 1$. This implies that we run the following regression:

$$
\hat{q_{C,t}} - \hat{q_{s,t}} = \omega^{A^S} \hat{A_{S,t}} + \omega^{A^C} \hat{A_{C,t}} + \omega^m \hat{M}_t + \omega^{F^S} \hat{F}^S_{\rho,t} + \omega^C \hat{C}_t,
$$

where the ω's are regression coefficients that map into the log-linearization coefficients of equation (23) as stated in the main text in Eq. (31).

III.d Untargeted data momemts

The data for Table 3 covers the period from 1999 Q1 to 2019 Q4. All statistics are for the HP filtered business cycle component.

We download the real personal consumption expenditures series from FRED (Federal Reserve Bank of St. Louis). This series is in billions of chained 2012 dollars and seasonally adjusted. We express this series in per capita terms. To get the per capita time series, we divide the real GDP series by the real GDP per capita series in billions, both series downloaded from FRED. Then we take logs and apply the HP filter. We use the HP-filtered real GDP per capita series to calculate the business cycle correlations. We define investment as described in the calibration Section 4.2. It is the real gross private domestic investment series, expressed in billions of chained 2012 dollars divided and per capita terms. Then we take logs and apply the HP filter.

We calculate leverage for S-banks using data from Compustat, defining firms as S-banks as described in Section 4.2 in paragraph "parameters to match moments of the ergodic distribution". Book leverage is defined as the ratio of total liabilities (ltq) to total assets (atq). Market leverage is defined as the ratio of total liabilities (ltq) to the market value of assets, defined as the sum of the market value of equity (cshoq*prccq) and total liabilities (ltq). We apply the HP filter to each series and calculate its standard deviation and business cycle correlation. To calculate the market leverage rate for C-banks, we use Compustat/CRSP data in addition to BHC data to get the market value of equity for the subset of publicly traded BHCs.

We define the data counterpart of S-bank liquidity provision using Flow of Funds data as the sum of money market mutual fund shares (Table L.206), repurchase agreements not involving commercial banks or the Fed (the total from Table L.207 less repos by the Fed and commercial banks), and financial sector commercial paper (Table L.209). We measure total liquidity provision as the sum of shadow bank liquidity provision and commercial bank liquidity provision, the latter defined as total deposits of BHCs.

We define the yield C, yield S, and the liquidity benefit in the data as described in Section 4.2. That is, we use the deposit rate BHCs pay on deposits for yield C, and the AA rated financial commercial paper series downloaded from FRED for yield S. We use the option-based measure of the riskfree rate without a liquidity premium as calculated by Van Binsbergen, Diamond, and Grotteria (2019) to calculate a liquidity premium. Note that the option based riskfree rate time series is slightly shorter, starting in 2004 Q1 and ending in 2018 Q1. We map the spread between the rate on S-bank and C-bank debt to the spread between the AA-rated financial commercial paper series and deposit rates.

III.e Simulation Data Variables

For our post-crisis simulation exercise, we download quarterly data for the period from 2008 to 2018.

We measure bank-dependent sector output (BDS output in Fig. 2) by applying the share of bank-dependent sales (saleq) from Compustat to the real GDP per capita series from FRED, Federal Reserve Bank of St. Louis. We follow the definition in Kashyap, Lamont, and Stein (1994) to classify firms as bank-dependent if they do not have a S&P long-term credit rating. Because mortgages make up the largest share of the bank loan portfolio, we also add construction and real estate firms as identified by SIC codes 6500-6599 (real estate), 1500-1599 (construction), and 1700-1799 (construction contractors, special trades) to the set of bank dependent firms. We consider all other firms as bank-independent. We measure investment of the bank dependent sector (BDS investment) as the ratio of capital expenditures (capxq) to assets (atq) from Compustat using the same definition of bank dependent firms. We define consumption as the quarterly time series of real personal consumption per capita, in chained 2012 dollars, downloaded from FRED, Federal Reserve Bank of St. Louis.

We define aggregate liquidity as the sum of shadow bank debt and commercial bank liquidity provision. We define the shadow bank liquidity supply as the sum of money market mutual fund shares (Table L.206 in the Flow of Funds), repo (Table L.207) less the repo position of the Fed and banks, and commercial paper from the domestic financial sector (Table L.209). We define the commercial bank liquidity supply as deposits using the sum of total deposits of BHCs. We then express these time series in chained 2012 dollars and in per capita terms. We calculate the S-bank debt share as the ratio of the shadow bank liquidity supply as defined above in total liquidity provision.

The shadow bank leverage time series comes from Compustat data using SIC codes to define shadow banks. Shadow banks are GSE and Finance companies (27%) with SIC codes 6111-6299 (excluding SIC codes 6200, 6282, 6022, and 6199), REITS (66%) with SIC code 6798, and Miscellaneous investment firms (4%) with SIC codes 6799 and 6726. We measure leverage as the value weighted total debt over asset ratio. This means that each quarter we sum up total liabilities and total assets of all financial institutions that meet our shadow bank definition. Leverage is then just the ratio of total liabilities to total assets for each quarter. The commercial bank leverage series is derived similarly using also Compustat data. We define commercial banks as financial institutions with SIC codes from 6000 to 6089 or SIC code 6712.

III.f Parameter Sensitivity Checks

Table B presents the results of the model if a single parameter is changed relative to the benchmark calibration of Section 4. In the first three columns, we focus on parameters of the liquidity function (19).

First, we perturb the scale of the liquidity benefit ψ; as one would expect, higher ψ raises liquidity production (line 16) and convenience yields (lines 10–11). As a result, deposit rates for both types of banks decline (lines 8–9), the banking sector expands, and it funds more productive capital (line 1). Because the marginal utility from liquidity is higher, S-banks increase leverage (line 5). Overall, the economy suffers higher deadweight losses from bank failures of both kinds of banks, partially the effect of higher GDP on consumption (line 17). Higher ψ exacerbates the implicit subsidy to C-banks from deposit insurance and thus increases the C-bank market share.

Column (2) perturbs the weight on S-bank liquidity α. Predictably, higher α, leads to an expansion in the S-bank share (lines 2-3). Raising α increases the wedge between decentralized equilibrium and the optimal planner allocation; in other words, the S-bank sector expands by less than it ideally would for this increase in α. This reduces overall liquidity production (line 16), which raises convenience yields (lines 10–11). The capital stock, but also S-bank leverage and defaults, increase.

In column (3), we vary ϵ, which parameterizes the elasticity of substitution between S-bank and C-bank debt. The main effect of higher ϵ is a smaller S-bank sector, as households care less about the composition of liquidity services and C-bank have a competitive advantage.

We do not include variations in γ_H in Table B since the effect of this parameter is discussed at length in Section 5.2 of the main text. Overall, our take-away from these liquidity parameter variations relative to the baseline is that they affect model moments in predictable and sufficiently distinct ways that allow for separate "identification" of the parameters' values when calibrating.

In column (4), we vary dispersion of S-banks' idiosyncratic productivity shocks. An increase in this parameter makes S-banks riskier at the same level of leverage. As a result, S-bank debt becomes more expensive, and S-banks reduce leverage (line 5), yet not by enough to prevent a higher default probability (line 13). Lower leverage implies that their equity is less attractive, causing a somewhat smaller S-bank share. The level of σ_{ρ^S} is a key parameter for the effect of increased capital requirements: the riskier S-banks are in the model, the less the economy benefits from shifting intermediation activity away from C-banks to S-banks.

In column (5), we consider variations of the S-bank bailout probability π^B. If S-banks do not receive any guarantees of their liabilities as in the simple model of Section 3, they choose 13% lower leverage than in the benchmark model (line 5). Their capital share rises, yet their debt share declines. Increasing π^B by only 1.5pp relative to the benchmark has large opposite effects on the S-bank leverage (+5.31%) and defaults (+169.25%). This comparison demonstrates that π^B has large and non-linear effect on the behavior of S-banks, and is a key parameter for determining their leverage choice.

Table C evaluates different specifications of the liquidity function described in the main text, equations (A1) and (A2). In these functions, the relative weight that S-banks receive in liquidity production depends directly on their default risk: greater S-bank defaults reduce their liquidity benefit. This specification nests the function used in the main text, (19), as special case with $\nu = 0$. As we can see, the net effect of these changes is similar to a reduction in α, but with a quantitatively smaller effect than the direct reduction in α considered above in Table B. In fact, we verified that our baseline model with a reduction in α by 5% yields very similar aggregate moments to the model in column 2 of Table C. The reason is that time-variation in the liquidity benefits produced by S-bank debt makes this debt less attractive for households unconditionally. As result, households substitute to C-bank debt, which leads to a smaller S-bank share of capital and debt. This comparison demonstrates that our preference specification is flexible enough to accommodate a more direct interaction between default risk and liquidity premia; however, for a reasonable S-bank default rate level and volatility, this interaction is sufficiently captured by the level of α.

Table B: Parameter Sensitivity Checks

	(1) ψ		(2) α		(3) ε		(4) σ_{ρ^S}		(5) π^B	
	-25%	$+25\%$	-25%	$+25\%$	-25%	$+25\%$	-5%	$+5\%$	$= 0$	$= .865$
Capital and Debt										
1. Capital	-2.57	2.47	-0.83	0.74	0.05	-0.05	-0.03	0.01	0.38	-0.26
2. Debt share S	4.44	-2.30	-29.89	31.32	3.03	-3.34	1.50	-1.46	-8.10	0.99
3. Capital share S	6.56	-3.90	-28.88	29.89	2.93	-3.22	0.25	-0.29	1.37	-2.38
4. Capital S	3.82	-1.52	-29.46	30.84	2.98	-3.27	0.22	-0.28	1.75	-2.60
5. Leverage S	-3.19	2.53	-0.88	0.59	0.06	-0.07	1.83	-1.70	-13.27	5.31
6. Leverage C	-0.01	-0.01	0.02	0.03	0.03	-0.01	0.00	-0.00	0.02	0.02
7. Early Liquidation (runs)	-4.18	3.49	-1.32	0.95	0.16	-0.19	1.75	-1.62	-12.77	4.95
Prices										
8. Deposit rate S	11.75	-9.98	3.52	-2.96	-0.17	0.22	0.39	-0.19	-12.28	11.57
9. Deposit rate C	15.15	-14.01	4.47	-3.64	-0.26	0.29	0.20	-0.12	-2.44	1.53
10. Convenience Yield S	-22.49	20.50	-6.65	4.43	0.32	-0.40	-1.97	1.97	13.89	-3.07
11. Convenience Yield C	-18.71	17.48	-5.40	3.55	0.31	-0.34	-0.27	0.25	3.46	-1.97
12. Corr(Conv. Yield C,Y)	-17.29	8.75	4.24	-10.18	2.10	-2.50	-1.28	1.23	8.40	-13.82
Welfare										
13. Default S	-43.08	50.34	-13.52	9.90	0.64	-0.81	-12.32	14.26	-94.65	169.25
14. Default C	-1.76	1.23	0.11	1.43	0.83	-0.40	0.04	-0.02	0.90	0.54
15. GDP	-0.19	0.18	-0.06	0.05	0.00	-0.00	-0.00	0.00	0.03	-0.02
16. Liquidity Services	-4.00	3.44	8.95	-4.98	-0.38	0.43	0.50	-0.48	-3.81	1.40
17. Consumption	-0.022	0.015	0.000	-0.006	-0.001	0.001	0.000	-0.001	0.031	-0.047
18. Vol(Liquidity Services)	17.82	-11.78	1.51	1.03	-0.59	0.48	2.54	-2.49	-13.33	-14.73
19. Vol(Consumption)	0.52	-0.39	-0.99	1.85	0.17	-0.13	0.07	-0.05	-0.68	5.46

This tables presents moments of the simulated model for different single-parameter changes. In columns (1)-(3), we decrease or increase the parameter by 25%. In Column (4), we de-/increase the volatility of S-banks' idiosyncratic shock by 5%. In Column (5), we set the bailout probability of S-banks to zero or increase it to 86.5%. All numbers are percentage changes relative to the baseline.

Table C: Time-varying S-bank Liquidity Preference Function

	(A1) with $\nu = 1$	(A1) with $\nu = 10$	(A2) with $\nu = 1$
		Capital and Debt	
1. Capital	-0.01	-0.09	-0.02
2. Debt share S	-0.37	-3.60	-0.93
3. Capital share S	-0.36	-3.49	-0.88
4. Capital S	-0.37	-3.57	-0.90
5. Leverage S	-0.00	-0.05	-0.05
6. Leverage C	-0.00	-0.01	-0.01
7. Early Liquidation (runs)	0.01	0.07	-0.23
		Prices	
8. Deposit rate S	0.03	0.35	0.08
9. Deposit rate C	0.05	0.47	0.11
10. Convenience Yield S	-0.06	-0.64	-0.14
11. Convenience Yield C	-0.06	-0.57	-0.13
12. Corr(Conv. Yield C,Y)	0.42	3.91	-2.98
		Welfare	
13. Default S	-0.13	-1.38	-0.29
14. Default C	-0.10	-0.39	-0.20
15. GDP	-0.00	-0.01	-0.00
16. Liquidity Services	0.08	0.82	0.22
17. Consumption	0.000	0.001	0.000
18. Vol(Liquidity Services)	-0.07	-0.59	22.84
19. Vol(Consumption)	-0.02	-0.16	-0.05

This tables presents moments of the simulated model for different specifications of the utility function for liquidity, see Section 4.4 in the main text. All numbers are percentage changes relative to the baseline. Columns 1 and 2 use specification (A1), which means that S-bank liquidity supply in the liquidity aggregator H is multiplied by $(1 - F^S)^\nu$. The first column sets $\nu = 1$, and the second column sets $\nu = 10$. The third column instead uses specification (A2) that also incorporates S-bank run risk and bailout probability.